

Open Research Online

The Open University's repository of research publications and other research outputs

Developing Primary Teachers' Mathematics Subject Knowledge: A Practitioner Research Study that Explores the Developed Nature of Primary Teachers' Subject Knowledge in Mathematics; the Factors Which Influenced its Development and its Interrelationship with and Influence on Changes in Professional Practice, Within the Context a Mathematics Specialist Teacher Programme (MaST)

Thesis

How to cite:

Morgan, Deborah Catherine (2016). Developing Primary Teachers' Mathematics Subject Knowledge: A Practitioner Research Study that Explores the Developed Nature of Primary Teachers' Subject Knowledge in Mathematics; the Factors Which Influenced its Development and its Interrelationship with and Influence on Changes in Professional Practice, Within the Context a Mathematics Specialist Teacher Programme (MaST). EdD thesis The Open University.

For guidance on citations see [FAQs](#).

© 2016 The Author



<https://creativecommons.org/licenses/by-nc-nd/4.0/>

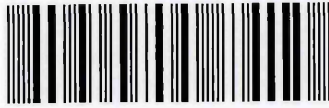
Version: Version of Record

Link(s) to article on publisher's website:

<http://dx.doi.org/doi:10.21954/ou.ro.0000ef8e>

Copyright and Moral Rights for the articles on this site are retained by the individual authors and/or other copyright owners. For more information on Open Research Online's data [policy](#) on reuse of materials please consult the policies page.

oro.open.ac.uk



Developing Primary Teachers' Mathematics Subject Knowledge

A practitioner research study that explores the developed nature of primary teachers' subject knowledge in mathematics; the factors which influenced its development and its interrelationship with and influence on changes in professional practice, within the context a Mathematics Specialist Teacher Programme (MaST)

Debbie Morgan

March 2016

Final Submission for the Award of Doctorate in Education

DATE OF SUBMISSION: 28 OCTOBER 2015

DATE OF AWARD: 23 MARCH 2016

ProQuest Number: 13834789

All rights reserved

INFORMATION TO ALL USERS

The quality of this reproduction is dependent upon the quality of the copy submitted.

In the unlikely event that the author did not send a complete manuscript and there are missing pages, these will be noted. Also, if material had to be removed, a note will indicate the deletion.



ProQuest 13834789

Published by ProQuest LLC (2019). Copyright of the Dissertation is held by the Author.

All rights reserved.

This work is protected against unauthorized copying under Title 17, United States Code
Microform Edition © ProQuest LLC.

ProQuest LLC.
789 East Eisenhower Parkway
P.O. Box 1346
Ann Arbor, MI 48106 – 1346

Abstract

The study is set within the context of the Mathematics Specialist Teacher Programme (MaST) at a University in England. This programme was a two year Masters' level professional development for primary teachers. The research questions explore the nature of the subject knowledge developed by the teachers; the factors which influenced its development and how the developed subject knowledge interacts with and influences classroom practice.

A practitioner research approach was adopted, influenced by the concept of phenomenography. Grounded theory was applied to analyse the data in order to develop theory. A mixed methods strategy was used, involving mainly qualitative data with some quantitative data collected through both questionnaires and group interviews.

The main findings were that:

The development of teacher subject knowledge is dynamic and continuous and has the potential to develop within the context of practice.

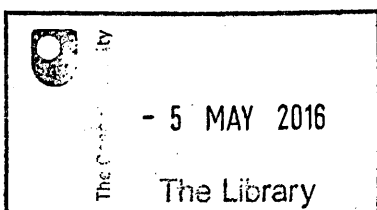
A connected structure applied to the programme design had significant impact on teacher development.

The particular constructs used within the programme acted as vehicles for transference of developed subject knowledge into practice.

My recommendations for policy and future research are:

The development of a national framework for primary teacher subject knowledge in mathematics, which outlines the required knowledge and recommendations for development. This would require research, agreement and evaluation.

Further research into effective strategies for the development of teacher subject knowledge in mathematics, including the application of a big ideas framework



DONATION
T 372.72 2015
Consultation CBU

Contents

Abstract.....	2
Chapter 1: The Introduction	8
The Research.....	8
The Researcher	8
Primary Teachers’ Subject Knowledge.....	10
The Mathematics Specialist Teacher (MaST) Programme	10
Chapter 2: The Literature Review	15
Section 1: The nature of teacher subject knowledge	15
Section 1: The nature of teacher subject knowledge	16
Primary teachers’ subject knowledge, a cause for concern	16
The nature of mathematics subject knowledge	18
Profound fundamental knowledge of mathematics.....	19
Connective knowledge.....	20
Instrumental and relational knowledge.....	21
Structural knowledge of mathematics.....	23
Duality of Knowledge	26
Making Connections within the learning process.....	28
Generality and knowledge of representations	29
Representational Knowledge.....	29
Pedagogic Content Knowledge	30
Summary of the Literature Review Section 1	32
Section 2: The Development of Teachers Subject Knowledge	33
The process of change	34
Characteristics of Effective Professional Development	36
Summary of the Literature Review Section 2	40
Section 3: Key theoretical constructs that underpin the context my research	41
Mathematical Thinking	41
Representation.....	44
Proportionality	46
Generalisation.....	47

Pattern	47
Summary of the Literature Review Section 3	48
My research questions.....	48
Question 1: What is the nature of subject knowledge developed by teachers engaged in the professional development programme?	49
Question 2: How does teacher knowledge develop and what factors influence its development?	49
Question 3: In what ways does the development of subject knowledge interact with and influence the practice of teachers?	49
Chapter 3: The Methodology and Research Methods.....	51
The Research Questions.....	52
Research Methods and Methodology	53
Methodology.....	53
Practitioner Research.....	53
Phenomenography.....	56
Research Methods	59
Research Instruments	61
Questionnaires.....	61
Interviews.....	64
Analysis	69
Grounded Theory.....	71
Issues of validity and reliability of quantitative and qualitative analysis	78
Triangulation	82
Ethical Considerations.....	83
Summary of Research Methodology and Methods	84
Chapter 4: The Findings	87
Findings from the questionnaire data	87
Findings from the interview data.....	92
Answering the research questions.....	93
Question 1: What is the nature of subject knowledge developed by teachers engaged in the professional development programme?	93
Relational Knowledge	94

Connective Knowledge	95
Deep Knowledge	96
Pattern and Generality.....	96
Representational Knowledge.....	97
Mathematical Thinking	99
You don't know what you don't know.....	99
Summary of Findings in relation to Question 1	100
Question 2: How does teacher knowledge develop and what factors influence its development?	101
Summary of the findings in relation to Question 2.....	103
Question 3: In what ways does the development of subject knowledge interact with and influence the practice of teachers?	103
Talk.....	104
Questioning.....	105
The nature of subject knowledge developed influences the nature of practice.....	105
The 5 big ideas form a structure for the development of subject knowledge and pedagogy.....	106
Connective knowledge impacts on practice	106
Knowledge of pattern and generality impacts on practice.....	107
Knowledge of representations impact on practice.....	108
Summary of the findings in relation to Question 3.....	108
Sustainability of Change.....	108
Summary of Chapter 4.....	110
Chapter 5: Discussion, Analysis and Development of Theory	111
Research Question 1: What is the nature of subject knowledge developed by primary teachers engaged in the professional development programme?.....	111
Deeper not Higher.....	111
Listening Knowledge	114
Representational Knowledge.....	116
Generality.....	123
Research Question 2: How does teacher subject knowledge develop and what factors influence its development?	124
Subject knowledge develops as a way of thinking.....	125
The Influence of the big Ideas and making connections	126

Development is experiential and reflective	127
Networking.....	129
Research question 3: In what ways does the development of subject knowledge interact with and influence practice?.....	129
Subject knowledge and pedagogy are bound together.....	130
Teachers subject knowledge develops within the act of teaching	133
Pedagogic vehicles for the transformation of subject knowledge into practice	135
Concluding Remarks.....	138
Chapter 6: The Conclusion	139
Key findings and new knowledge	139
The nature of teacher subject knowledge	139
The development of teacher subject knowledge	141
Recommendations for practice in relation to the development of teacher subject knowledge	145
The interrelationship of subject knowledge and professional practice	146
Recommendations for practice.....	148
How the findings of my study fits into the current educational context.....	149
Looking Forward: Recommendations for Policy	150
Looking Forward: Recommendations for Future Research	151
Looking Forward: Dissemination of my research	153
References	154
Appendix 1a The Northampton MaST Programme Connective Model.....	170
Appendix 1b The constructs used in the Northampton MaST Programme to transfer the five big ideas into professional practice	170
Appendix 2 The Questionnaire	172
Appendix 3 The Interview Questions.....	174
Appendix 4 Example of Coding – Theme Representation	175
Appendix 5 Ethical Code	179
Appendix 6 Themes that emerged from the interview data	182
Figure 1 Example of task to develop and assess structural thinking Mason et al. (2009 p1	
Figure 2 Tag Cloud Word Frequency Search in NVIVO 9 (QSR 2010)	74

Figure 3 Summary Word Frequency Search in NVIVO 9 (QSR 2010)75

Figure 4 Creating Nodes (Codes) **Error! Bookmark not defined.**

Figure 5 Coding Audio Files.....77

Figure 6 Question 4 Mathematical Development.....89

Figure 7 Question 5 Impact on Teachers’ Practice90

Figure 8 Question 6 Areas of mathematical development.....91

Figure 9 $5 \times 3 = 15$ 121

Figure 10 Cyclical and Dynamic Nature of the Development of Teacher Subject Knowledge135

Chapter 1: The Introduction

This chapter provides an introduction to the research in terms of the researcher, the context of the research and relevant background information.

The Research

The research is set within the context of a two year teacher professional development programme known as the Mathematics Specialist Teacher Programme (MaST). A key element of the programme is the development of teacher subject knowledge in mathematics. The research seeks to understand the nature of the knowledge developed by teachers engaged in the programme; how such knowledge develops and its interaction with and influence on their professional practice. The research takes the form of a practitioner research investigation, where I am the practitioner facilitating the teacher professional development. The adoption of practitioner research provides the opportunity to engage in deep reflection and analysis of my own practice and how it impacts on the development of teacher subject knowledge. I feel immensely privileged to be in the position to work with the experienced and committed professionals engaged in the programme and to research the development of their subject knowledge and related practice and in turn develop insight into my own practice as a provider of teacher professional development.

The Researcher

My career in education has spanned twenty six years. I trained as a generalist primary teacher, have taught every year group within the primary phase and have taken on school leadership roles including headship. I have also worked for a local authority as a primary mathematics advisor and at a university as a senior lecturer in mathematics education working with trainee teachers. I subsequently led the regional MaST programme which was based at the university where I worked and forms the context for my research. At the time of submitting my thesis I have moved from the university into the national role of Director of Primary Mathematics at the National

Centre for Excellence in Teaching Mathematics (NCETM). Within this role I provide advice and expertise to the national government through the Department for Education in England and am involved in the shaping and implementation of national policy regarding primary mathematics. Within this role I ensure that I continue to remain engaged in teacher professional development and work regularly with teachers.

My own interest in mathematics and the development of my expertise has taken place over the years since becoming a primary teacher. I entered the profession, as most teachers do, with the statutory 'O' level (subsequently GCSE) in mathematics. Teaching mathematics sparked an interest in the subject and made me want to improve my own subject knowledge. Through study at both degree and masters' level with the Open University over many years, my subject knowledge has developed. This had opened the door to a whole new world of mathematics which I still find exciting and has had a considerable influence on my professional practice. It was my desire that teachers engaged in my MaST programme should have a similar experience.

My role within the MaST programme for the period in which the gathering of data took place was that of Programme Director of the Northampton MaST programme. I had overall responsibility for the programme across six universities and nineteen local authorities. It was however of personal importance to me that I not only directed and managed the programme but that I also taught on it. Teaching not only provided the enjoyment and satisfaction that is important to me as a practitioner but it also provided insight that informed the decisions I made at a managerial level. The interplay of these two elements continue to be important to me and even within my current role as Director of Primary Mathematics (NCETM) I continue to teach teachers on a regular basis and occasionally children. Within the Northampton MaST Programme I was the main tutor for some of the teachers (between 30% and 50%) who were enrolled directly with Northampton University, but I also had the opportunity to teach all of the teachers at shared residential events

which included tutors and teachers from all six universities and took place at the start of the programme.

Primary Teachers' Subject Knowledge

Primary teachers in England are generalist teachers and typically teach the full range of the curriculum, teaching up to ten subjects including mathematics. Most teachers in England enter the profession through a one year post graduate programme, having studied a three year degree course, which for the majority was focused on an arts related subject. Only 3% of teachers have studied any mathematics within their degree (Williams 2008) and within initial teacher training programmes there is limited time available for study of the subject. Continued professional development (CPD) in mathematics is variable between teachers and location, and many teachers receive little or no additional training beyond their PGCE year (Williams 2008).

The Mathematics Specialist Teacher (MaST) Programme

The Mathematics Specialist Teacher (MaST) Programme was a national initiative which included strategies to address primary teacher subject knowledge and forms the backdrop for the research. This programme arose from a key recommendation from the Williams Review (2008), a review of the teaching of Mathematics in English Primary Schools and Early Years Settings commissioned by the UK Government. A key recommendation stated that *there should be at least one Mathematics Specialist in each primary school, in post within 10 years, with deep mathematical subject and pedagogical knowledge* (p7). These specialist teachers were to be developed from existing experienced professionals teaching in primary schools in England. The programme was a part time two year Masters Level programme with most of the face to face training conducted outside of school hours, including weekends.

Invitations to tender for delivery of the programme went out to Higher Education Institutions (HEI) across England in early 2009. Eight of the tenders were successful and several involved more

than one university. I led the tendering process for a submission involving six universities which was successful. I subsequently became the director of this particular programme and took on a key leadership role. Central government allocated particular regions to each of the eight providers defined by local authorities. The programme that I led was responsible for nineteen local authorities. It was known as the Northampton MaST programme as it was The University of Northampton who held the contract with central government. Although six universities were involved in my programme it was my aim that we should work together and that the programme would emerge as one programme and not six individual programmes.

The aims of the national MaST Programme were set by central government and emerged from the Williams Review (2008), these were to develop deep subject knowledge, deep pedagogic knowledge and coaching and mentoring skills of participating teachers. Other requirements were also set nationally in terms of the overall content and delivery of the programme; however the detail of both content and delivery was the responsibility of individual providers.

The national requirements were that teachers should receive 30 hours of face to face training from a HEI institution with the provision of half termly local network meetings led by local authorities but informed by the HEI institution and the content should cover the mathematics concepts of the primary curriculum (DfEE 1999) and how these concepts developed through the key stage three curriculum (DfES 2004). Within these parameters the Northampton MaST programme team, consisting of the MaST leaders from each of the six universities in the joint tender, developed the detail of the content and the order and structure of how it was delivered. Although developed through discussion within the programme team, I was programme's director of the Northampton MaST programme and thus the details of the programme of necessity reflect my values as a practitioner and their underpinning theoretical and conceptual constructs.

My research has been designed to allow me to gain deep insight into the impact of my practice on teacher professional development, to influence my future practice, make recommendations for the practice of others and inform future policy. Although, as detailed above, there were several areas the programme was tasked with developing, the area I was particularly interested in was that of the development of teacher subject knowledge in mathematics.

Central to the development of my own subject knowledge has been the development of a depth of understanding of mathematical structures and the ability and desire to make connections between mathematical ideas. Askew et al (1997) identified that the most effective teachers of primary mathematics have *a rich network of connections between different mathematical ideas* (p3). In order to facilitate teachers to increase their ability to make connections in mathematics a structure for the Northampton MaST programme was created around five big ideas in mathematics education. The hypothesis was that these would enable teachers involved to look at mathematics in a different way, at the macro level and view the whole of the mathematics curriculum together under each of the ideas, thus providing greater opportunities to make connections. The ideas selected were mathematical thinking, pattern, representation, proportionality and generality. These ideas were agreed by the team of university tutors involved in the programme as appropriate to form the basis of the programme. Each of these ideas is discussed in detail within Chapter 2 of this thesis.

A focus on just a few key ideas studied across two years provides the time and opportunity to revisit the ideas regularly. The hypothesis was that regularly revisiting would enable a recursive elaboration of conceptual understanding as discussed by Davis and Simmt (2006) and provide the basis for the development of deep and sustainable knowledge. Central to the process of teacher development was practitioner reflection, particularly within the context of practice and there was the requirement for teachers to revisit the five big ideas regularly within their daily teaching of

mathematics and record comments in a reflective journal. It was anticipated that this regular reflective element would help to embed the ideas and, develop deep knowledge which impacts on practice,

To facilitate the integration of teacher development into professional practice a set of pedagogic and learning constructs (Mason & Johnston- Wilder 2004) were selected by the programme team to be used in conjunction with the five big ideas to support the process of the development of teacher subject knowledge. Our intention was that teachers should also use them in the development of pupil subject knowledge. We anticipated that these constructs would further facilitate the making of connections in mathematics. The constructs and their interrelationship with the five big ideas are outlined in Appendix 1a and 1b. Their intention was to support teachers in making connections in mathematics and transfer their learning into professional practice. The final column of the table (Appendix 1b) outlines the use of four pairs of learning constructs referred to as *mathematical powers*, a term used by Mason (2005), who built on the work of Polya (1957). They in particular facilitate the development of *mathematical thinking*, opportunities to recognise *pattern* and extract generalisations and are elaborated further in Chapter 2.

The coherent structure applied to the programme though the constructs outlined above was designed by myself and the programme team to support constructivist theories of learning as developed by Piaget, Vygotsky and Bruner, as discussed in Mason and Johnston Wilder (2004). The structure invited learners to engage in autonomous mathematical thinking, construct and make sense of mathematics for themselves through cognitive and social interaction, all of which is scaffolded and facilitated by the teacher. The items in the structure are not intended to be a definitive list of relevant constructs, but rather to provide the opportunity for regular engagement and deepening of knowledge. The defined structure of the programme was also intended to

provide a shared language in which to both think about and articulate subject knowledge. The importance of the ability to articulate conceptual understanding is identified by Ainley and Luntley (2007).

My interests and the focus of my research lie within the context of the development of teachers' subject knowledge through the Northampton MaST Programme outlined above. However I recognise that it is likely that any focus on teacher subject knowledge is going to be enmeshed with teachers' pedagogic practice and hence consideration will be given to the interplay between subject knowledge and practice. As a practitioner seeking to support the improvement of teachers' subject knowledge I am seeking to improve my understanding of the nature of the knowledge that is required for effective teaching of the subject within the primary school context in order to improve my professional practice. The next chapter which forms the literature review includes an exploration of the existing literature in relation to the nature of subject knowledge relevant to the primary teacher, including its relationship to making connections in mathematics as discussed above. It also considers how this knowledge is developed, features which influence its development and the complexity of teacher change; an understanding of which are relevant to my practice if it is to be effective. Lastly the chapter explores the five big ideas and related constructs outlined above as these are likely to influence the development of teacher subject knowledge and its impact on their practice.

Chapter 2: The Literature Review

The focus of my research is on the development of primary teachers' subject knowledge in mathematics and is set within the context of my professional practice as a provider of teacher professional development as outlined in Chapter 1. I wish to develop greater understanding of the nature of the subject knowledge necessary for effective teaching of primary mathematics; how such knowledge develops, and finally its interaction with and influences on teachers' professional practice. The overall aim of my study is to improve my own practice in supporting the development of teachers' mathematical subject knowledge. This chapter aims to explore existing literature relevant to my study and I have selected four areas which I consider necessary to inform my research. These are outlined below with a brief explanation of why each is important and relevant to my study.

Section 1: The nature of teacher subject knowledge

If my own practice is to be effective in developing teachers' mathematics subject knowledge then it is important that I understand the insight that has already been generated about the nature of the knowledge required by teachers for effective teaching. Such understanding will influence my practice within the setting of practitioner research and also support my analysis of the teachers' developing subject knowledge within the context of my study.

Section 2: *The development of teacher's subject knowledge*

The purpose of teacher professional development is to bring about change, changes to knowledge, changes to practice and sometimes changes to beliefs. I anticipate and indeed aim for teacher change to occur within the context of my study. Here I aim to identify within extant literature how such change is seen in relation to the development of teacher subject knowledge and to seek to understand how others perceive such knowledge developing and the factors which influence it, including features of effective professional development. The literature explored in this section will both inform my practice within the context of my study and also support my

analysis of the data relating to the factors which influenced the teacher professional development that takes place within the context of my practice.

Section 3: Key theoretical constructs that underpin the context of my research

The context for my research is the Northampton MaST Programme. As discussed in Chapter 1 five big ideas provide the structure of the programme. This final section will explore the literature concerning each of these ideas to gain a deeper understanding of their potential influence on learning mathematics. It is reasonable to assume that these constructs will influence the process of teacher development and the nature of the subject knowledge developed.

The literature review draws on literature from across the world. The term primary mathematics is used in England to describe learning within the age range of 5 to 11, however in other countries the term elementary is used to reference to the same or similar stage of learning. The two terms primary and elementary are used interchangeably throughout this thesis.

Section 1: The nature of teacher subject knowledge

This section of the literature review seeks to identify what is already known about the nature of the knowledge necessary for effective teaching of primary mathematics. There is widespread agreement (Ofsted 2013, Ofsted 2008, Williams 2008) that primary teachers' subject knowledge requires improvement. Necessary to the process of improvement is identification of the knowledge that is required. There is however is no agreed framework as to the nature of the subject knowledge required for effective teaching of primary mathematics. In order to support the improvement of teachers' subject knowledge within the context of my professional practice I am seeking to understand the nature of the subject knowledge necessary for effective teaching of mathematics within the primary school context.

Primary teachers' subject knowledge, a cause for concern

The mathematical subject knowledge of primary teachers has for many years been seen as a cause for concern and various means have been sought to address this perceived weakness. Alexander et al. (1992) in a discussion paper published by the Department for Education and Science (DES) identified the need to improve primary teachers' subject knowledge. This resulted in revised requirements for initial teacher training as set out in circular 10/97 (DFEE, 1997). This document placed greater emphasis on subject content knowledge and specific mathematics subject areas were listed for study, alongside the requirement for initial teacher training providers to audit student teachers knowledge of the mathematical concepts listed. Currently trainee teachers are required to pass a professional skills test in *numeracy* prior to the commencement of training (DfE 2015). Although initial teacher education establishments are required to develop teacher subject knowledge (DfE 2014) the current standards that teachers have to satisfy to be awarded qualified teacher status have moved away from a specific list of content. Standard Fourteen for Qualified Teacher Status (QTS) (2008 to August 2012) stated that teachers should *have a secure knowledge and understanding of their subjects/curriculum areas and related pedagogy to enable them to teach effectively across the age and ability range for which they are trained* (Q14 S1:9). The current standards state that teachers should *have a secure knowledge of the relevant subject(s) and curriculum areas, foster and maintain pupils' interest in the subject, and address misunderstandings* (DFE, 2012 p6). Neither of these standards explain the nature of the subject knowledge required for teaching primary mathematics and the list of subject knowledge areas identified in circular 10/97, as will be seen through the discussion in this chapter, do not necessarily capture the nature of the subject knowledge required for teaching primary mathematics.

The Ofsted report *Made to Measure* (Ofsted, 2008) which reported on inspection evidence of mathematics teaching highlighted the necessity to further *promote enhancement of subject knowledge and subject-specific teaching skills in all routes through primary initial teacher*

education. It would seem that the strategies put in place; including the teaching and auditing of higher level mathematics within initial teacher training has not had the desired impact and primary teachers' subject knowledge in mathematics remained a concern, as evidenced in subsequent reports from Ofsted (Ofsted, 2013).

Whilst the government recognised weaknesses in primary teachers subject knowledge, so did the research literature that will be examined below. However where the research literature is not in line with government analysis is in the nature of the weakness and the nature of mathematics knowledge necessary for effective teaching of the subject. The research literature recognises teacher subject knowledge as being more complex than a list of content could express and the solutions to the problem are portrayed as requiring a more sophisticated approach.

The nature of mathematics subject knowledge

One solution proposed to address weak subject knowledge in primary teachers' has been to require higher mathematics qualifications of prospective teachers. Many teacher training providers have raised their requirement from a minimum qualification of grade C at GCSE to grade B for student teachers entering training. However research (e.g. Askew et al., 1997, Carpenter et al., 1988) indicates that subject knowledge as reflected in formal qualifications, such as grade B at GCSE or even A' level mathematics does not necessarily translate into the type of knowledge required for effective teaching of mathematics in the primary school. The research of both Askew et al. (1997) and Carpenter et al. (1998) illustrate that there is no correlation between the qualification standard attained by teachers and effective teaching, higher qualifications do not by default translate into more successful teaching of the subject within the primary setting. This does not necessarily mean that subject knowledge is unimportant for primary teachers, but rather that it may be a particular type of subject knowledge that is required. The nature of which is explored in the sections below.

Profound fundamental knowledge of mathematics

Ma (1999) carried out a comparative study of teacher subject knowledge from a sample of teachers in the US and China who taught elementary mathematics. Her findings concluded that the Chinese teachers' subject knowledge was more effective than that of the US teachers in terms of enabling pupil progress despite the teachers having less formal schooling and fewer higher level qualifications. However she does not draw the conclusion that subject knowledge is unimportant. A closer scrutiny of the research shows that the Chinese teachers did indeed have good and effective subject knowledge but it was of a different nature than that of teachers in the US. An example from the research which illustrates the difference between the Chinese and the US teachers is that, although US teachers were able to state required formulae for area and perimeter, they had insufficient depth of understanding to analyse the mathematics underpinning the formulae and were unable to identify and address pupil misconceptions. The Chinese teachers demonstrated a deeper understanding of the mathematics, which was exhibited in their ability to analyse the mathematics in generalised forms, extracting the underlying structures of the mathematics that formed the formulae. A clear link between Chinese teachers' subject knowledge and pedagogy was seen when having analysed the mathematics they were able to identify multiple and appropriate representations to develop in pupils a deep conceptual understanding of the mathematics. Ma describes the nature of this knowledge as *profound fundamental understanding of mathematics* (PFUM) (Ma, 1999).

A more recent comparative research study of mathematics learning in schools in China and the US (She, 2011) studied pupils' conceptual understanding and like the teachers in Ma's research found that it was superior in China. The study observed that in the Chinese schools mathematics was viewed as a network of interlinking ideas that could be applied to multiple contexts and as in Ma's research the ability to generalise mathematical structures featured strongly.

Connective knowledge

The importance of connecting ideas in mathematics and indeed seeing the whole of mathematics as a connected web of ideas features strongly within the research literature, for example see (Barmby et al., 2009, Rowland et al., 2009, Iannone and Cockburn, 2008, Askew et al., 1997). Askew et al. (1997) in their enquiry into what makes a teacher of primary mathematics effective, categorised styles of teaching into three orientations, a transmission orientation, a discovery orientation and a connectionist orientation. They concluded that the most effective teaching came from teachers of a connectionist orientation. These teachers had a form of subject knowledge which made connections between different ideas in mathematics and sought to expose those connections in their teaching. Furthermore embedded in Ma's (1999) PFUM is a form of connective knowledge. For example Ma (1999) discusses how Chinese teachers connect the four operations of addition, subtraction, multiplication and division and are interested in the relationship between them, using this knowledge to develop both deep conceptual understanding and quick and efficient ways of calculating. Ma argues that since primary mathematics is focused on the four arithmetic operations, these connections form a "roadway" through the curriculum.

Iannone and Cockburn (2008) also viewed mathematics as a web of interconnected ideas and in their research identified teachers who made connections between mathematical ideas in their teaching as those who were the most effective in developing pupils' conceptual understanding and their ability to generalise mathematics. Barmby et al. (2009) highlighted the value of making connections in order to develop the way learners think about mathematics. As discussed above in the two comparative pieces of research between the US and China (She, 2011, Ma, 1999) viewing mathematics in generalised forms and seeing it as a network of connections was a feature of

effective teacher knowledge and pupils' learning. Others (Nunes et al., 2009c, Baroody et al., 2003, Gray et al., 1999) also argue for the importance of knowledge of a well-connected web of mathematical ideas.

It would seem that emerging from the research literature is a strong sense of the importance of the development of a connective form of knowledge by teachers to support effective teaching of mathematics.

Instrumental and relational knowledge

The writing of Richard Skemp (1976) on subject knowledge in mathematics is significantly cited within the research literature in terms of providing insight into the nature of subject knowledge in mathematics. Skemp (1976) developed the use of the terms instrumental and relational understanding of mathematics as a way of describing two different but necessary forms of knowledge relevant to learning mathematics. He defined instrumental understanding as knowledge of mathematics techniques and procedures which when applied allow the learner to arrive at a correct answer. Relational understanding he defined as a deeper understanding which gets beneath the surface of techniques and algorithms and exposes the mathematical structures and relationships which created them and hence why they work when used to solve a problem. He also explored whether a focus on instrumental understanding is sufficient in the teaching and learning of mathematics and what advantages a focus on relational understanding might have. The advantages he identifies in terms of instrumental understanding is that it is easier and quicker to teach, as pupils learn to carry out procedures step by step to arrive at correct answers and can therefore achieve immediate results. Relational understanding takes a greater time to develop. However whilst he acknowledges the benefits of instrumental learning; he concludes that relational learning has distinct advantages over instrumental learning for the following reasons:

- It supports application of mathematics to a new task and can guard against misapplication; for example adding zero to the end of a decimal number when multiplying by ten.
- It is more sustainable in terms of memorisation and recall due to its initial construction within the context of making connections and generality.
- It can develop a higher interest level, since it is focused on why something works and therefore is more engaging and motivational to learn.
- It encourages and develops independent thinking and may encourage the learner to explore related lines of mathematics due to their own interest and motivation.

He concludes that instrumental understanding may have a role within short term periods of a child's education but over the whole period relational understanding is essential for effective and sustainable learning.

Skemp (1989) places an emphasis on relational knowledge as understanding why and a key part of that is making connections between symbols and concepts, understanding the mathematics behind the symbols. However he does also acknowledge the importance of making connections between concepts and *that learning mathematics involves learning not isolated facts, but a connected knowledge structure* (1989 p155). He places an emphasis on relational understanding being a connective form of knowledge and relates relational to understanding the relationship between the task, the content and the larger matrix of mathematical knowledge. Skemp's (1989) analysis of subject knowledge in mathematics complements the arguments for a connective form of subject knowledge explored so far. His key contribution to the nature of this knowledge, I would argue, is the need to understand *why* techniques, rules and procedures are used in mathematics in order to connect them to other related techniques and procedure, which in turn makes the learning of mathematics easier. An example he provides is, knowing that the formula for finding the area of a triangle is equal to $\frac{1}{2}$ base x height. Whilst this fact can be assimilated

quickly and then applied to any triangle, an instrumental understanding isolates this fact to one context, that of finding the area of a triangle. However if an understanding of why this rule works is developed then it can be adjusted and applied to more than just triangles. The areas of other polygons can be deduced, such as rectangles, parallelograms and trapeziums. Within Skemp's (1986) critique, connections are made where mathematical techniques rules and procedures are understood at a relational level which includes an understanding of *why* they work. He provides a good rationale for this form of connective knowledge. Seeing different rules, procedures or techniques as part of a connected whole enables them to be remembered more easily, cuts down on the amount of mathematics to learn and deepens understanding such that learning is more sustainable over time.

Structural knowledge of mathematics

Mason et al. (2009) make use of Skemp's (1986) term *relational* knowledge, and interchange it with the term *structural* knowledge of mathematics. The term structural places a particular emphasis on the form that this relational knowledge takes. They argue that there is a need for the development of structural thinking to support understanding in mathematics and provide several practical examples in their research of the form this knowledge takes and how it might be developed within teaching. One such example is:

Think about the following mathematical sentence:

18 +

Box A

= 20 +

Box B

a) Can you put numbers in Box A and Box B to make the three correct sentences like the one above?

b) When you make a correct sentence, what is the relationship between the numbers in Box A and Box B?

c) If instead of 18 and 20 the first number was 226 and the second number was 231 what would the relationship between the number in Box A and Box B?

d) If you put any number in Box A can you still make a correct sentence? Please explain your thinking carefully

Mason et al. (2009 p15)

Figure 1 Example of task to develop and assess structural thinking

Mason et al. (2009) note that those who were successful in completing the task with confidence, and who were able to explain their thinking clearly, went beyond seeing a pattern, and focused their attention on the structure of the mathematics. They again argue that developing understanding of structure reduces the reliance on memory; and further notes that structural thinking enables connections to be made and supports relational understanding and generality. They recognise the demands that this type of learning makes on teachers' subject knowledge and pedagogy. The teacher needs both structural knowledge of the mathematics themselves and also the recognition that this type of understanding cannot be acquired by pupils through a transmission approach to teaching. A transmission approach is defined as when the teacher *tells* pupils what to do. Instead a structural approach requires a more skilful methodology of engaging pupils to attend to structure and to make their own connections thus developing their own thinking and understanding, which is in line with a constructivist approach to teaching and learning as developed by Vygotsky (1978), Bruner (1966), Piaget (1941) and discussed in Mason and Johnston Wilder (2004). Mason et al. (2009) argument for a structural form of knowledge is in line with Skemp's (1986) notion of relational knowledge and this is reinforced through their interchange of the words structural and relational. There is however, I would argue, a slightly different emphasis in the use of the word structural where the connections that are being made are within the one element of mathematics and not across elements. The problem is solved in the example above by looking at the relationships within the one structure where there is a relationship between one side of the equation and another. This relationship however is generalised to other examples in the same form.

Within Mason et al (2009) structural knowledge is the notion of making connections, in the example above between one side of an equation and the other in order to solve a problem. The connections exemplify the interdependent relationships between one side of the equation and the other in line with the structure of the whole equation.

In addition to Mason et al (2009) another consideration of the notion of structure within mathematics is presented by Ball and Hill (2009). They argue that teachers need not only to be able to do the mathematics, (that is to act instrumentally Skemp's (1986)) but also need to understand how mathematics is constructed (a similar interpretation to Mason et al (2009) of relational understanding). They argue that this type of knowledge impacts on pedagogy and supports teachers in recognising helpful strategies and representations to support learners in seeing structures and constructing understanding of concepts for themselves. They cite the example of Mrs Gonzales who lacked structural understanding and when attempting to teach subtraction of a negative numbers told the children that the subtraction sign and the negative sign combine to make a positive, thus

$$-1 - (-3) = 2$$

Although carrying out Mrs Gonzales' instructions would lead to a correct answer, pupils would be unable to understand why the method worked. In Skemp's (1986) words they would have gained instrumental knowledge but not relational knowledge. The structures that create the relationships are not explored and conceptual connections are not made. It is argued that although this might result in the children being able to "do the mathematics" but this form of knowledge alone does not support the advancement of conceptual understanding.

Carpenter et al. (2003), carried out a longitudinal study of the development of elementary pupils' mathematical thinking. Structural understanding of arithmetic was identified as important on the part of the teacher, it was recognised that pupils develop this knowledge by making their own connections in terms of the structure of the mathematics. The approach they took to teaching and learning emphasised structures and relationships within arithmetic and supported pupils in making structural connections for themselves; through particular types of pedagogy. For example a problem such as:

$$63 + 24 = \square + 25$$

was solved by pupils recognising first of all the equals sign as a symbol meaning equivalent and then the structure of the relationship between the two sides of the equation, that 25 is one more than 24 and so the unknown has to be one less than 63 in order to maintain the equivalent relationship. As with Mason et al. (2009) the interplay between subject knowledge and pedagogy was crucial in supporting pupils in constructing these connections for themselves. The approach that was taken led to a more flexible and relational understanding of the mathematics and supported later understanding of algebra.

It would appear that structural knowledge is embedded within the concept of relational knowledge as developed by Skemp (1986) and making connections features strongly. Connective knowledge involves recognising structures and relationships which form the structures of the mathematics and provides the means for connections to be made.

Duality of Knowledge

Tall and Gray's (1994) research concerns the importance of a deeper form of knowledge akin to the connective, relational and structural knowledge discussed thus far. They developed the idea of *proceptual* knowledge, a knowledge which exemplifies a duality of understanding of both process and concept within the domain of mathematics. An example of this is where an expression such as $3+2=5$ can be seen both as a process to be executed in adding 2 to 3 to reach 5, but also as a concept in terms of the relationship between two and three and its equivalence to 5. They argued that the ability to move between these two ideas, something which is both static and dynamic, is characteristic of a successful mathematical learner who has the flexibility to move between different ideas within the one representation.

A similar duality in understanding mathematics is echoed by Sfard (1991) who argues that there is the need to conceive mathematics *in two fundamentally different ways: structurally - as objects, and operationally - as processes* (p1). She argued that *the processes of learning and of problem-solving consist in an intricate interplay between operational and structural conceptions of the same notions* (p1). Sfard (1991) uses the word structural to express the structure of the mathematics in terms of something that is static which can be viewed as an object which embeds meaning, in terms of existing concepts. Her emphasis is on the interplay between structures and operations in mathematics and argues that through operating on objects, structural understanding might be developed. In Mason et al.'s (2009) use of the term structure they place the emphasis on the relationship between objects which is a connective form of knowledge in which reasoning comes into play. This reasoning leads to relational understanding, whereby clarity in terms of the structures which form the relationships is gained. Both Mason et al. (2009) and Sfard (1991) express the idea of a deeper form of knowledge of mathematics where generality is abstracted.

Tall and Gray's notion of duality is similar to Sfard (1991). However Sfard places an emphasis on the complementary relationship between mathematics as objects (described as concepts by Gray and Tall 1994) and mathematics as processes and uses the metaphor *two sides of the same coin*. Although she recognises that understanding of processes will develop first, as seeing abstract mathematics as objects is much harder, the end goal is that they should be seen together. This is where her notion of duality differs from that of Tall and Gray (1994), they place the emphasis on moving between the two aspects with flexibility, whereas she places the ultimate goal of seeing the two aspects together.

The connective form of knowledge discussed by Sfard (1991), Tall and Gray (1994) and Mason et al. (2009) places an emphasis, not on the connections between or across mathematical concepts, but on the connections within mathematical concepts, for example the concept of addition within

the structural form of an equation is both an operation and an expression of a relationship between numbers. These two elements are of necessity connected within the structure of the equation. These two forms of connections both within and between mathematical concepts are important in developing deep conceptual subject knowledge.

Making Connections within the learning process

Making connections in mathematics as described in the sections above is an ongoing process and needs to happen at many levels. Smith et al. (1993) identify that knowledge is of necessity built up in pieces over time. However advances in learning happen when that knowledge comes into conflict with another piece of knowledge and there is discord between the old knowledge and the new knowledge. In this context knowledge must be adjusted, reconciled and connections made. I have observed teachers whose only concept of multiplication is repeated addition and when asked to consider multiplication of fractions there is discord because their model of repeated addition does not seem to work, it is not possible to have multiple groups of something that is reducing in size. Another model of multiplication is required, that of scaling where the object is enabled to increase (scaling up) and decrease (scaling down). The two models of scaling and repeated addition then need to be reconciled. It can be argued therefore that connections are necessary both between concepts and within concepts.

Izsak (2008) observed that a person who uses one piece of knowledge in one context may fail to use the same piece of knowledge in another context, where, from an observers' point of view, it would make sense to do so. Connections are not made automatically, but are built up over time and reflect the security, and developing fluency of the mathematics. The research by Orrill and Brown (2012) observes that teachers can know aspects of mathematics, but fail to connect and bring those aspects together in the dynamic interplay within the classroom setting. The disposition to both look for and make connections is important for teachers' subject knowledge.

Generality and knowledge of representations

Knowledge of useful representations and the ability to generalise and make connections has already been mentioned within the research discussed so far (She., 2011, Ma., 1999, Askew et al., 1997, Shulman., 1986). It is also mentioned by Williams (2008) as necessary for teacher knowledge. Golding et al. (2002) also argue for the need for primary teachers to have knowledge of representations upon which to draw in their teaching and also knowledge of algebra to support their ability to generalise. They also hold the viewpoint that all of these things cannot be adequately addressed at the training stage but are more effectively addressed within the context of Continuing Professional Development (CPD). These two ideas will be discussed further in Section 4 as they form two of the five big ideas.

Representational Knowledge

Shulman (1986) talked of the importance of representations within the context of making the subject *comprehensible to others* (p9), and includes it within his categorisation of pedagogic content knowledge, which is discussed below. Lesh et al. (1987) claim that the ability to generate a variety of mathematical representations is integral to mathematics teachers' subject knowledge, Ball (1990c) asked 19 pre- service teachers to generate contexts to illustrate

$$1\frac{3}{4} \div \frac{1}{2}$$

Seventeen of the teachers could carry out the calculation, but only five were able to represent the calculation within a context. Turner (2008) further observed that the use of representations in effectively teaching mathematics was frequently identified as an area of weakness for beginning teachers. The representations the teachers in their research selected were often based on superficial qualities, such as their attractiveness rather than deeper aspects, such as how they might support the development of conceptual understanding. Another aspect of teacher representational knowledge is not only to be able to represent concepts in different ways to

support pupil learning, but also to recognise concepts presented in different ways themselves. Izsak (2008) studied teachers understanding and ability to represent multiplication of fractions and identified that teachers lacked the ability to recognise distribution when presented in different ways, which was inhibiting children's progress in conceptual knowledge. A further aspect was that they sometimes failed to recognise pupils' representations of concepts and were thus unable to respond to their thinking and the connections they had made. Thus it seems that representational knowledge is an area for teacher development.

Pedagogic Content Knowledge

The final form of subject knowledge discussed in this section is pedagogic content knowledge (PCK). This is a particular form of subject knowledge identified originally by Shulman (1986) and deemed to be necessary to effective teaching. It incorporates elements of subject knowledge already discussed, in particular structural knowledge, representational knowledge and relational knowledge. Through the construct of PCK Shulman (1986) takes the discussion of teacher subject knowledge beyond a traditional interpretation of knowledge of the facts and techniques of the subject and recognises that teacher knowledge needs to be characterised by an understanding of the structures from which the subject is constructed and argues that the teacher need not only understand that something is so; the teacher must further understand why it is so. This is akin to Skemp's (1976) relational understanding, knowing not only the 'how' (instrumental understanding), but also the 'why' (relational understanding). Shulman's notion of PCK relates to a connective form of knowledge; however his discussion is in the main generic, encompassing all subjects and not specifically mathematics and therefore does not go into the detail of conceptual mathematical connections as discussed by others authors mentioned earlier.

Shulman's (1986) use of the term PCK is an attempt to capture the essence of the interrelatedness between subject and pedagogic knowledge. He defined PCK as that *special amalgam of content*

and pedagogy that is uniquely the province of teachers, their own special form of professional understanding.....it goes beyond knowledge of subject matterto the dimension of subject matter knowledge for teaching (Shulman 1986 p9). He includes within the category of PCK teachers' understanding of and ability to select useful forms of representations of concepts and an awareness of common misconceptions. These two aspects of teacher knowledge featured also in Ma's research (Ma, 1999), as characteristics of the deeper understanding exhibited by the Chinese teachers.

PCK is not simply a bridge between subject and pedagogic knowledge, but a distinct type of subject knowledge within the domain of teacher subject knowledge. The notion of the concept of PCK seems to have proved useful in stimulating debate and has been widely and further developed by other researchers and writers (for example Hill and Ball., 2009, Rowland et al 2009) . Rowland et al developed a framework, called the knowledge quartet, for categorising teacher subject knowledge within the context of teaching. The four constructs of their framework, identified as key in relation to teachers' subject knowledge, are foundation; transformation; connection; and contingency knowledge. Their knowledge quartet included features of PCK, such as teachers' use of representations, identifying misconceptions and making connections in mathematics.

Ball (Ball et al., 2008, Ball, 1993,) also built on Shulman's(1986) work but this time with a specific focus on mathematics . Through studying mathematics teaching they sought to develop a more detailed understanding of the nature of the pedagogic content knowledge (PCK) required for effective teaching of mathematics. Two sub categories of PCK were identified as important: knowledge of content and students and knowledge of content and teaching. They also identified a form of *specialised* content knowledge, the nature of which is specific to the teaching of mathematics and extended teacher knowledge into other realms, such as the notion of *Horizon*

Knowledge. Horizon knowledge is defined by Ball et al. (2008) the ability to make connections and see the bigger picture in terms of mathematical knowledge. Ball et al. (2008) stresses the importance of the teacher knowing and understanding what is *on the horizon* that is the mathematics that will be learnt in the future. Horizon knowledge allows the teacher to make connections with and appreciate how that later knowledge must impact on teaching and learning in the present (Loewenberg Ball, 2008, Ball, 1993). Thus the ideas of connectionist teaching in mathematics are taken further to include making connections to the mathematics in the future and the conceptual journey of the learner.

Summary of the Literature Review Section 1

The above section of the literature review has attempted to explore the nature of the knowledge required by teachers for the effective teaching of mathematics within the primary sector. I have sought to identify a definition of teacher knowledge of which my research will seek to analyse and develop greater understanding. It recognises that the required knowledge for effective teaching is not knowledge necessarily represented by traditional qualifications such as A' level mathematics (Askew et al. 1997), but is knowledge which goes deeper than this and does not covering the same content. Ma (1999) described the knowledge required as profound understanding of fundamental mathematics (PUFM). Such knowledge is characterised by a view of mathematics as a web of interconnected ideas from which pupils can make conceptual connections (Askew et al. 1997, Barmby et al., 2009, Iannone and Cockburn, 2008, Ma, 1999, Rowland et al., 2009, She, 2011). It is structural in nature (Mason et al., 2009, Shulman, 2004, Gray and Tall, 1994) in the sense that exposing the underlying mathematical structures enables relational understanding to be developed (Skemp, 1976) and connections made. It connects subject and pedagogic knowledge and the interrelated nature is expressed in teachers' ability to support pupils in making connections and developing structural and relational knowledge (Mason et al., 2009). Teachers' ability to identify and use representations to develop conceptual

understanding is seen as important (Williams, 2008, Rowland et al., 2009, Ma, 1999). The ability to seek and identify generality in mathematics is also seen as necessary (Williams, 2008, Goulding et al., 2002).

The theme of making connections in mathematics is strong within the research literature and runs through all of the literature discussed above. These connections are made at various levels in terms of teachers' subject knowledge. There are connections to be made within mathematical structures, for example the relationship between two sides of an arithmetic equation (Carpenter et al., 2003), connections between aspects of a concept, for example the formulae for finding the area of a rectangle, a triangle, a parallelogram and a trapezium (Skemp 1986), making connections between concepts, for example scaling and multiplication; connections between abstract mathematics and real life (Ball 1990c); connections between pupils' current knowledge of mathematics and its relevance to mathematics they will learn in the future (Ball 1993). As a consequence of the literature reviewed here the Northampton MaST programme placed making connections at its centre. Thus in my study it will be valuable to analyse the importance that teachers place on this construct in terms of the development of their subject knowledge.

Section 2: The Development of Teachers Subject Knowledge

The research is not only seeking to extend understanding of the nature of subject knowledge necessary for teaching primary mathematics but also to develop insight into how subject knowledge develops, what factors influence its development and what synergies there are with teachers' practice. Necessary to understanding this process is a comprehension of current knowledge concerning teacher change, factors that influence such change and features of effective professional development that bring about change. My discussion includes a consideration of changes to teachers' beliefs about mathematics in conjunction with changes to knowledge and practice.

There is a suggestion within the research literature that development of subject knowledge in mathematics may impact not only on changes to practice but also changes to teacher beliefs regarding the subject. For example Skemp (1989) argued that teachers whose subject knowledge is based on instrumental understanding will teach in a different way from those whose knowledge is based on relational understanding and their beliefs about the teaching of mathematics will be different. Ball argues that beliefs are influenced by experience (Ball, 1990a) and thus the experience of learning mathematics and developing subject knowledge within the programme, may impact on beliefs and practice. In Ball's study, as teachers engaged in the development of their subject knowledge and experienced a different way of learning mathematics and tried out different ways of teaching it, their beliefs about mathematics changed. Consequently the literature concerning the beliefs and their effect on practice will also be important in my understanding of the teacher development process.

Due to the large amount of research available within the field of educational change and teacher professional development, I will, in the main, focus on that within the field of mathematics teaching.

The process of change

Many writers acknowledge that the educational change process is complex and difficult to achieve (Fullan, 2001b). The National Numeracy Strategy, initiated in England in 1999 was one of the largest initiatives of its kind and aimed to bring about change in the subject knowledge and practice of thousands of teachers across England. The external evaluators of the Numeracy Strategy acknowledged that bringing about change is complex and there are many influencing factors involved (Fullan and Earl, 2002). Guskey (2002) called for those engaged in the professional development of teachers to be aware that change is a gradual and a difficult process.

Houssart (2000) analysed the impact of changes to practice with regard to the use of resources to support calculation after one year of the National Numeracy Strategy and associated professional development and concluded that some teachers had made significant changes to their practice whilst for others changes were limited or superficial. Anghileri (2006) found similar results even after five years. Fullan (2001a) argues that teachers will not make changes to practice unless there is a moral purpose; that is they see it as imperative to children and their learning. The crucial question is how to instigate change and make it sustainable? Most evaluations of professional development programmes are carried out immediately on completion and no follow up is made (Franke et al. 2001). Franke et al (2001) carried out a follow up study of a programme focused on the development of mathematical thinking. They found that all the 22 participating teachers had maintained changes to their practice and ten had continued to grow. Characteristics of those who continued to grow were a belief in the importance of mathematical thinking, a commitment to its development in their practice and interaction with likeminded teachers.

There seem to be two main views with regard to the process of change and the relationship between beliefs and practice. The first is that it is necessary to change teacher beliefs and attitudes in order to influence changes to practice and the second is that changes to practice need to come first and the resulting changes to pupil outcomes will influence changes to attitudes and beliefs. Both of these viewpoints are discussed by Guskey (2002) who argued that most recent research supports the second view that changes to beliefs and attitudes are preceded by changes to practice and pupil outcomes. Swan and Swain (2010) confirm this second view and in their research asked practitioners engaged in their professional development programme to suspend beliefs and make changes to their practice. This was supported by a reflection process to bring about changes to attitudes and beliefs. The teachers within their research experienced a

transformation from a transmission or discovery view of teaching to a connectionist one as a result of seeing its impact on practice.

As indicated above teacher development and sustainable change are complex processes and several authors consider not only factors that facilitate change, but also factors which inhibit change (see for example: Goos and Geiger, 2010, Boylan, 2010, De La Cinta Munoz-Catalan et al., 2010, Guskey 2002, Fullan 2001b). Back et al. (2009) identified external testing, perceived curriculum constraints and lack of support from their schools as barriers to teacher change. Other teachers questioned in the same research identified pupil resistance to change as an inhibiting factor, pupils used to one particular style did not respond well to an alternative approach.

Goos and Geiger (2010) argued that no one can change teachers, teachers change themselves. They claim that change takes place within the context of creating opportunities for teachers to change themselves. This would indicate that change is different for each individual. Liljedahl (2010) identified that not all change is the same in nature even though teachers may have shared the same experience. Different prior experiences and contexts at the time of the CPD can influence the nature of the change that takes place. Similarly Mason argues that *I cannot change others, but I can work at changing myself* (Mason, 1994).

Characteristics of Effective Professional Development

Whilst the change process is difficult and complex, as outlined above, the research literature does identify key characteristics of professional development that have been shown to bring about effective and successful change. There are several research studies where CPD has been evaluated and there is recognition of the influence of teacher reflection (Warren, 2009, Zwart et al., 2007, Garet et al., 2001, Loucks-Horsley et al., 2003,). Clarke and Hollingsworth (2002) argued for a structure which is cyclical and allows for continual teacher reflection. This is echoed by

Darling- Hammond and McLaughlin (1995) who argued that teacher CPD needs to go beyond acquisition of knowledge and skills to critical reflection on their own practice. Warren (2009) identified the opportunities for reflection as one of the key factors to the success of a CPD programme where the focus was on the development of algebraic thinking. Tunks and Weller (2009) also identified the collaborative nature and shared reflection of their CPD programme as supporting successful outcomes in the development of teachers' mathematical thinking.

The reading of research literature can be a valuable means of stimulating teacher reflection. De Geest (2011) explored the use and impact of the utilisation of research and identified five positive outcomes that resulted from its use: it provided teachers with new ideas, developed greater depth in terms of teachers' thinking, teachers felt better informed, developed greater confidence and were given authority in their arguments. She also found that teachers believed that the use of research within the context of professional development contributed to its quality. Research however is not always used with the context of professional development, not all of the teachers within De Geest's research had interacted with research, although a sizable proportion had, this contrasts with Hargreaves' research (1996) where there was limited use or no engagement with research by teachers. Many advocate (for example Swan and Swain (2010), Warren (2009), Guskey (1986)) that CPD should be informed by theory and research and indeed involve elements of research on the part of the participants. They believe that this supports critical and deep reflection which in turn supports the development of teacher knowledge.

A valuable context in which teacher reflection might take place is within of the process of teacher collaboration as identified by Preciado-Babb and Liljedhal (2012). Where teachers collaborate there is the opportunity for teacher reflection and the embedding and refinement of ideas. Opportunities for collaboration can be provided through the establishment of communities of practice (Wenger, 1999). A community of practice can occur when a group of people gather,

normally in person, but also virtually (see for example Rosenbaum and Shachaf, 2010). The community is characterised by common features within its membership, for example shared goals, shared values and common social or professional characteristics. (Wenger, op.cit.). These communities meet regularly over time and provide opportunities for shared learning, and the development of common characteristics, such as shared language. Important also is the potential development of a shared identity (Wenger, 1999). The context for such communities to develop is provided within the Northampton MaST programme. Teachers who are participating in the programme come together for the most part into relatively small groups (between ten and twenty) and which I would argue have the potential to become, in Wenger's interpretation communities of practice (Wenger, 1999). These communities have the potential to impact on teachers' development in two ways, firstly in a social constructivist sense (Vygotsky, 1978) where learning is developed in a social setting. This is more than just teachers learning from each other but as Rensnick (1991) claims meaning is constructed through socialisation rather than instruction. She argues that thinking and understanding are actively developed by the collective rather than the individual; that the social interaction influences the way in which knowledge is constructed. She quotes Vygotsky (op.cit.) and Mead (2009) who argue that social experiences can shape the kinds of interpretive processes available to individuals. This leads to joint construction of knowledge.

Other factors that have been shown to support effective mathematics professional development include the practical engagement of teachers as both teachers and learners of mathematics. Within this process a focus on mathematical thinking has been found to be significant. The research by Steinberg, Empson and Carpenter (2004) identified that a focus on the development of mathematical thinking was a key factor in the change process and the effectiveness of their CPD programme. This is also supported by Warren (2009) Swan and Swain (2010). The research by Orrill and Brown (2012) focused on teacher subject knowledge in mathematics. They identified

three key strategies to develop subject knowledge. Firstly the necessity to build new coherent knowledge alongside the ability to apply that knowledge to a wider context; secondly to provide support for teachers in organising their existing knowledge into a coherent framework and thirdly careful selection of mathematical representations that help teachers to identify the salient points of concepts and also link to and illuminate their procedural knowledge.

The importance of experiential learning and teacher engagement in mathematics is also identified as important by Ball (1990a). She advocated that where change is to occur the methodology of the development needs to be experiential. Teachers or prospective teachers must experience for themselves a different way of thinking about mathematics. Elementary teachers often enter training devoid of a meaningful understanding of mathematics (Ball 1990a) due to the nature of the mathematics teaching that they experienced. They therefore need a different experience not only to develop their understanding but also to change their beliefs about the nature of mathematics and how it should be taught. Whilst learning mathematics experientially they both learn mathematics and learn about learning mathematics, both aspects being crucial for effective teaching. Noh and Webb (2015) argue that recent research into effective student learning of mathematics should be applied to teacher learning, providing the opportunity to experience learning mathematics, in a similar way to that of their students. They argue for an inquiry based approach where teachers are encouraged to make sense of the mathematics they are learning, working on and unpacking mathematical tasks to thoroughly understand the mathematics. This process is aimed at improving not only teachers' understanding of concepts and procedures but also how the mathematics might be represented in ways that aid learning. Loucks-Horsley et al (2009) suggest an immersion process, in this process teachers are immersed in learning mathematics within a pedagogic context that reflects the practices they are learning and developing. This they argue provides both experiences that enable them to learn content

knowledge and also to develop their own problem solving and process skills allowing them to understand how to assist their pupils' learning.

Another important feature in the development of teacher knowledge is that it needs to be directly linked to practice, where teachers are required to change or experiment on practice. Both Warren (2009) and Swan and Swain (2010) believe that changes to practice need to occur for effective CPD to be said to have taken place and applied this strategy in their research and CPD programmes. The link between the development of subject knowledge and practice are clearly important. Ball et al. (2009) discuss that effective CPD should integrate both subject and pedagogic knowledge since the interrelatedness is so significant, one cannot be developed without the other.

A final characteristic of effective professional development that I wish to draw attention to, is the time period in which the development takes place. Research by Garet et al. (2001) involved a large scale empirical comparison of the effects of different characteristics of professional development on teachers learning in mathematics and science. They identified that duration is important and longer programmes are more effective. They give two reasons for this; firstly longer programmes provide greater opportunities for in depth discussion of content and secondly they provide time for teachers to try out new ideas in their classrooms and engage in discussion regarding the outcomes.

Summary of the Literature Review Section 2

A primary purpose of professional development is to bring about positive change to teachers' professional practice which in turn improves outcomes for pupils. However the process of teacher change is complex and is influenced by many factors; such as knowledge, experience, beliefs (Ball, 1990a, Skemp, 1989) identity and practice (Wenger, 1999). Change is often difficult and the

literature identified potential inhibitors such as curriculum constraints, external testing and lack of school support as barriers to success (Back et al 2009). It was acknowledged that sometimes change can be superficial (Houssart, 2000), however strategies for deep and sustainable change through the process of professional development are identified in the literature. These include opportunities for teacher collaboration and the sharing and development of ideas together (Tunks and Weller, 2009). Communities of practice (Wenger, 1999) were identified as a powerful vehicle for collaboration. Teacher's engagement in critical reflection was seen as important (eg Warren, 2009) and key strategies identified to support this were collaboration and the reading of and engagement in research (Swan and Swain 2010). Other factors include the duration of professional development, with longer programmes noted as being more successful (Garet et al 2001). Specific successful aspects of mathematics professional development included engagement in mathematics as learners, engagement in mathematical thinking and the requirement trial changes to practice (Swan and Swain 2010).

Section 3: Key theoretical constructs that underpin the context my research

There are five key theoretical constructs, which underpin the professional development programme which provides the context for my research. These are known as the five big ideas within the programme and are referenced to in Chapter One. In seeking to understand the nature and development of teachers' subject knowledge it is likely that these key ideas will play at least some part in shaping their development. I will therefore outline the relevant academic and research literature that have supported my thinking.

Mathematical Thinking

Mathematical thinking is at the heart of learning mathematics, a claim made by many (e.g. Mason, 1982, Schoenfeld, 1992). Often an approach to learning which involves mathematical thinking is contrasted with a rote learning and memorisation approach. Schoenfeld (1992) compares the two approaches and identifies that mathematical thinking is about *seeking*

solutions, not just memorising procedures; exploring patterns, not just memorising formula; and formulating conjectures, not just doing exercises (p335). He also incorporates within his definition the ability to think flexibly, solve novel problems and analyse arguments put forth by others. He is not claiming that memorization and practice are not relevant, but that they should not be the only form of learning. Other research has explored the integration of factual, procedural and conceptual knowledge (Rittle-Johnson et al., 2002, Rittle-Johnson et al., 2001) and concluded that all play a role in successful learning.

Mathematical thinking is often allied with problem solving in mathematics. Schoenfeld (1992) intimates, that problem solving is an aspect of mathematical thinking, alongside, *core knowledge, problem solving strategies, effective use of one's resources, having a mathematical perspective and engagement in mathematical practices* (Schoenfeld, 1992 p335). Schoenfeld describes these as fundamental aspects of mathematical thinking. In designing the MaST programme, the original idea was to select problem solving as one of the five big ideas, but instead *Mathematical Thinking* was selected as it was believed to potentially include a wider brief and encompass a means of looking at the whole of the mathematics curriculum rather than just a section of it, which problem solving seems to be. Also as Schoenfeld (1992) points out, the term problem solving has received multiple interpretations and there is perhaps at present a lack of clarity as to what it means within the context of teaching and learning mathematics.

A key text on mathematical thinking is *Developing Mathematical thinking* by Mason et al. (1982). The authors closely align mathematical thinking with problem solving, which they interpret as a means to engage the learner with mathematical thought through the process of solving problems. Important in this process is *becoming stuck* which provokes the need to ponder, think again, and try other strategies in order to become *unstuck*. This engages the learner in mathematical thought which develops learning in mathematics and the ability to solve problems. Their work is

influenced by the work of Pólya (1957). Central to Pólya's (1968) notion of problem solving is conjecturing and reasoning. He argues that we secure our mathematical knowledge by demonstrative reasoning, but we support our conjectures by plausible reasoning (Pólya, 1968). He defines demonstrative reasoning as mathematics which is evident through applying reasoning of what we know and plausible reasoning as a means of guessing (conjecturing) what might be true, and is therefore provisional, but leads us on a journey of discovery to new learning. The ability to reason in mathematical ways is identified by Nunes et al. (2009b) as the most significant predictor of success in learning mathematics. If pupils have not developed the ability to reason mathematically by the end of Year 4 then they are unlikely to be successful in making good progress in the subject. Importantly in later research Nunes et al. identify that reasoning in mathematics is not a fixed ability but something that can be developed through teaching (Nunes et al., 2009a). The development of reasoning through mathematical thinking is incorporated within Mason's employment of *mathematical powers* (Johnston-Wilder and Mason, 2005, Mason et al., 1982). These are defined as natural human abilities, which when employed within the context of learning mathematics develop the ability to think mathematically, thinking mathematically is really about learning to use these powers in mathematical ways and in the exploration of mathematical problems (Mason et al., 1982). These powers are listed as:

- Specialising and Generalising;
- Conjecturing and Convincing;
- Imaging and Expressing;
- Sorting and Classifying (Johnston-Wilder and Mason, 2005).

Mason's construct of mathematical powers is utilized within the MaST programme that provides the context for my study.

Sfard (2008) considers the development of mathematical thinking within the context of communication and discourse. Within her classification of communication, she includes thought

alongside verbal expression and explores various vehicles, such as visual images and concrete objects to support communication. She also highlights the importance of shared discourse, where a reconciliation process occurs between one's own ideas and the ideas of others in making sense of mathematics. Thus she claims, participating in mathematical discourse is vital in the development of mathematical thinking.

Representation

Mathematics is an abstract subject which is represented in a variety of ways. A key purpose in representing mathematics is to communicate its essence. Sfard (2008) makes explicit reference to representations in the form of visual images and concrete objects to support thinking and communication of mathematics. Other forms of representations which support learning in mathematics are real life contexts, manipulable objects, pictures and diagrams, spoken language and written symbols (Lesh et al., 1987). They also make the claim that to understand mathematics one must be able to interpret and generate a variety of mathematical representations.

Representations can play a key role in solving mathematical problems and support the application of mathematical thinking. Lesh and Landau (1983) observe that real life mathematical problems are often solved by translating the problem into a representational form. For example the application of the bar model as a mathematical representational form has been particularly successfully in supporting pupils to model problems in a way which exposes the structures of the mathematics and enables pupils to see what techniques and procedures are applicable in order to solve the problem (Beckmann, 2004, Murata, 2008). Murata (2008) argues that if a certain representation is consistently used with instruction, this representation will become a part of students' mathematical thinking and a foundation for their future understanding. (p376).

Variation theory (Gu et al., 2004) is a key theory which underpins the teaching of mathematics in some countries and in particular is suggested as a reason why South East Asian countries who do

well in mathematics (Sun, 2011). One aspect of variation is the notion of conceptual variation which involves the use of multiple representations of the same concept to draw attention to the essence of a concept through highlighting those things that remain invariant and define a concept. The question *what's the same, what's different* as discussed by Mason et al (1982) is a useful construct to support the process of identifying invariant properties of a concept and as discussed in Chapter 1 is included in the Northampton MaST programme.

There is much in the research literature on the value of manipulatives to aid pupils' learning. Manipulatives are physical objects that can be handled and moved around (Montessori, 1964, Piaget, 2013 originally published 1941). Representations such as *Cuisenaire rods* or *Dienes blocks* can provide the opportunity to manipulate and explore mathematical ideas. Goutard (1964) a proponent of the work of Cuisenaire and Gattegno (1955) identified that paradoxically the use of the concrete materials allowed children to move much more quickly to symbolic representations as a result of the deep structural understanding that use of the rods developed and freed the learner from recourse to concrete materials, developing fluency and the ability to make connections.

Whilst there is much support for concrete manipulatives in the research literature, there is also some concern as to how readily learners abstract mathematical ideas from them. There are studies which concluded that the use of manipulatives have no benefit to learning mathematics (Resnick & Omanson, 1987; Thompson, 1992) McNeil and Jarvin (2007) consider that this can sometimes be due to insufficient expertise on the part of the teacher. For example where the manipulative selected is not the most appropriate for developing the concept, or the manipulative is simply selected to add interest or "fun" to the lesson (Moyer, 2001). Turner (2008) also identifies, that teachers often fail to sufficiently recognise that abstraction and the transference of conceptual understanding from concrete resources does not happen automatically through the

handling of the objects and the teacher has an important role to play in helping learners to make the necessary connections.

Representations do not need to be concrete but can be developed and used through the process of visualisation in mathematics. Giaquinto (2011) saw visualization as a means both to represent mathematics through imagination, and also as a tool to think with and explore mathematics.

Proportionality

Proportional relationships span much of the primary curriculum and are central to the teaching of fractions, decimals, percentages, ratio, multiplication and division, scaling and aspects of measurement etc. This big idea was to facilitate the teaching of these mathematical concepts and make connections between them. Teacher weaknesses in understanding proportional relationships, particularly fractions are well attested as a particular weakness of primary teachers' subject knowledge (Olmez, 2014, Kastberg et al., 2012, Orrill and Brown., 2012, Ball et al., 2008, Zhou et al., 2006, Ma., 1999, Ball., 1990c). Several specific points are identified:

- Teachers think in whole numbers and find difficulties in focusing on the relationship between two numbers at one time, for example the numerator and denominator within a fraction or the two parts of a ratio relationship (Ball, 1990c, Orrill and Brown, 2012, Loewenberg Ball, 2008)
- Teachers experience difficulties in Identifying proportional relationships within mathematics problems and therefore apply inappropriate strategies (Kastberg et al., 2012, Zhou et al., 2006)
- Teachers rely on algorithmic procedural knowledge and misremembered rules without the underlying conceptual understanding (Ball, 1990c, Zhou et al., 2006)
- Teachers fail to understand invariant relationships of ratio (Simon and Blume, 1994)
- Teachers have difficulties in distinguishing between proportional reasoning and additive reasoning (Orrill and Brown, 2012, Simon and Blume, 1994)

Generalisation

The act of generalising in mathematics is incorporated within the list of Mason's mathematical powers and the construct of *specialising and generalising*, developed originally by Pólya (1957) where specialising involves looking at one example, often a simple one. However the focus is not on answering one question, or solving one problem; the focus is to recognise mathematical structure and relationships and generalise to other related cases and instances. It is bound up with structural thinking and making connections, as discussed in section one of this literature review. Mason et al. (1982) notes that specialising is *seeing through the particular* by not dwelling in the particularities. He makes a distinction between *empirical* and *structural* generalisation. Empirical generalisation involves looking at several cases together and asking *what the same about them*. Drawing out the things that are the same and always will be the same are the generalities.

Structural generalisation is where the generality is derived from the mathematical structure of just one or a few cases, defining the generalisation as an essential property of the mathematics. This will often involve the process of conjecturing and convincing through logical reasoning. Generalisation is a necessary and powerful process in mathematics which deepens understanding. As referenced to in section one the ability to generalise is considered an important feature of teacher subject knowledge (Williams 2008, Ma 1999, Shulman 1986)

Pattern

The whole of the mathematics curriculum could be summed up in this one word of pattern. *Mathematics is the study of patterns abstracted from the world around us* (Ruth Lawrence quoted in DFEE, (1999)). Mathematics would not exist if it were not for pattern and the structure and relationships that it creates. Many mathematicians recognise the centrality of pattern to mathematics. Ian Stewart views the world as a *universe of patterns* (p1) (Stewart, 1995) and

Schoenfeld (1992) describes mathematics as the *science of patterns*. It is therefore important to develop teachers and in turn children's awareness of pattern and the structures it creates.

Pattern is closely linked to generalising and is often the first stage on the journey to a generalisation (Mason et al., 1982). The abundance of pattern within mathematics is *one of the most pleasing and satisfying aspects* of the subject (Mason, 1982 p74). Variation theory is posited on the value of recognising pattern and relationships which, which leads to deeper structural understanding of the mathematics (Gu et al, 2004). The exercises and activities that teachers devise for their pupils have the potential to draw attention to pattern and structure. This type of activity is not common in English teachers' teaching and potentially an area for development.

Summary of the Literature Review Section 3

The five big ideas have been shown to be important in the learning of mathematics and therefore important to teacher subject knowledge development. There is significant overlap between these five ideas, for example representations have the potential to support the development of mathematical thinking and recognition of pattern can lead to generality. Each have the potential to impact on teacher subject knowledge and it will be valuable to analyse the extent and nature of the development.

My research questions

The literature review has identified much material that is relevant to the context of my MaST professional development programme. It will be interesting to make comparisons with the outcomes of my programme. Section one has highlighted the complexity of identifying essential teacher knowledge for the effective teaching of primary mathematics. The features identified in section two in terms of effective teacher development have been incorporated into my programme, including the long duration of two years and it will be interesting to see if they are as effective within the context of the programme. Furthermore it will be interesting to observe the

potential relationship between the development of subject knowledge and teacher beliefs as discussed in section three. The literature in section three suggests that the five big ideas have the potential to be powerful features in the learning of mathematics and the development of subject knowledge.

I have identified three specific research questions that I believe will build on and deepen understanding of existing research and address my research aims.

Question 1: What is the nature of subject knowledge developed by teachers engaged in the professional development programme?

The research literature has identified that there is still no nationally agreed framework of what subject knowledge primary teachers need in order to effectively teach mathematics. It is hoped that my research will add to the existing body of literature and move forward the development of a National Framework. It is my intention to analyse the aspects of subject knowledge that teachers identify have developed and are supporting their practice as effective teachers of primary mathematics.

Question 2: How does teacher knowledge develop and what factors influence its development?

Whilst it is important to know what subject knowledge primary teachers need in order to effectively teach mathematics, it is of equal importance to understand how this subject knowledge might be developed. An analysis of the factors that influence teacher development will be important. There has been much less attention to this area within the research literature and I intend my research to add to the existing body of knowledge.

Question 3: In what ways does the development of subject knowledge interact with and influence the practice of teachers?

The interrelatedness of subject knowledge and pedagogy is a growing area of interest within the literature and is exemplified in Shulman's (1986) notion of PCK and other literature which builds on his ideas (eg Ball et al. 2008) and is seen as important to effective teaching of mathematics. Analysis of how teachers connect the two and how one influences the other should provide further insight into the relationship between the two within the context of professional practice.

Chapter 3: The Methodology and Research Methods

My study was set within the context of a new and innovative professional development programme which is described in detail in Chapter 1. The methodology however was not one of evaluation. I was not seeking to find out whether or not the programme was successful and whether the approaches taken were the most effective. Instead my research aimed to develop understanding of the phenomena that took place from the perspective of the receivers of the professional development programme. I sought to understand the development of teacher subject knowledge, what elements of the course did teachers recognise as impacting on their development and how did they perceive such development interacting with and influencing their own practice. I was aware that there were probably aspects of the professional development programme that were unsuccessful or could have been better; however these were outside the scope of my research. My research sought to understand **what** development teachers perceived had taken place in terms of their subject knowledge, **why** the development took place and **how** the development interacted with and influenced practice as reported by the teachers. Data was collected from three cohorts of teachers.

The first cohort of teachers commenced the programme in January 2010, the second in September 2010 and the third in September 2011. The fundamental structure of the programme did not change between these three cohorts of teachers. It continued to focus on the five big ideas and the other pedagogical and learning constructs that formed the structure of the content of the programme as outlined in Chapter 1. However two key changes were made to the organisational structure. The first was a change in the number of residential events. The first cohort had two residential events one at the start of year one and the other at the start of year two of the programme. Cohorts two and three were only offered one residential event at the start of their programme; however they received the same face to face time through additional day or half day events spread across the year. The rationale behind the changes was that the residential

event at the start of year one was important to provide sufficient time to immerse teachers in the key ideas that would be developed throughout the programme and to generate excitement and enthusiasm. By year two of the programme teachers were already familiar with the key ideas and had also formed important relationships with their key tutors, so it was decided to continue teaching in these established settings with tutors who knew the participants well and could build on previous learning.

The second change was to merge consideration of the five big ideas in year two of the programme but still maintain separate consideration in year one. Focusing on one key idea at a time was necessary in year one in order to develop depth and clarity of understanding, but less of a requirement in year two. Also it was felt valuable to merge them in year two and look at the links between them. This change was applied for cohorts two and three.

In this chapter I outline the research questions that were applied to my study and explain the research methods and methodology that was designed to answer them.

The Research Questions

These are the research questions that my research sought to answer and from which the methodology and research methods were shaped:

- What is the nature of the subject knowledge developed by teachers engaged in the professional development programme?
- How does teacher knowledge develop and what factors influence its development?
- In what ways does the development of subject knowledge interact with and influence the practice of teachers?

Research Methods and Methodology

Burton (2002) made a distinction between research methods and research methodology.

Research methods are the techniques used for gathering evidence; whereas the methodology addresses an analysis of, and justification for, the overall approach adopted. She argued that detailed consideration of the methodology should come first, and within that the values of the researcher should be exposed. I have attempted to follow this advice and consideration is given to the methodology first; including my values as a researcher and the type of researcher I am.

Methodology

The methodology encompassed two key elements, those encapsulated within the context of practitioner research and those underpinning the construct of phenomenography. I believe these two methodologies are complementary and drawing from both of these has served my research well. These two elements reflected my own values and beliefs regarding the nature and aims of my research.

Practitioner Research

My research was set within the context of practitioner research. I was a practitioner within the context of the professional development which formed the setting within which the research took place; the Northampton Mast Programme. My role was to plan the programme, including its structure and content and also to support its delivery, providing the opportunity to work with and interact with the teachers engaged in the programme.

A key consideration for anyone embarking on practitioner research is consideration of intent. Fox et al. (2007) identified that the overall intention of practitioner research should not be to change others, but instead to change oneself. As a practitioner involved in the professional development of teachers my role was to bring about change in others. However as a researcher my intent was

primarily to bring about change in myself, aided and motivated by the developed insight and understanding that emerged from the research process.

It has been argued (Cochran-Smith and Lytle, 1993) that practitioner research brings an important dimension to the field of research. Cochran-Smith and Lytle (1993) argued that practitioner research has the advantage of being conducted from the inside, providing insider insight, rather than the outside, as had traditionally been the case within educational research. Engaging in the context of the research both as a practitioner and a researcher had the potential to bring an element of insight that might not have been accessible to an outsider looking in. I was immersed in my own practice on a daily basis with regular interaction with the teachers engaged in the programme and so had opportunities to reflect on the actions I took to bring about change and facilitate the development of teacher subject knowledge. These actions were underpinned by theoretical constructs, which I have researched and developed a secure level of understanding. Sikes and Potts (2008) argued that the unique perspective that the insider brings to the research is of significant benefit in terms of existing knowledge and information they already have that would take an outsider a long time to acquire. They noted that the test of a good piece of research is the strength of the links between theory and practice. The amount of time engaged in the dual roles of practitioner and researcher has the potential to aid the transference between theory and practice. A key element of my research was the validation of the theoretical constructs which underpinned my professional practice, providing fresh insight and a deeper understanding of their impact. Sikes and Potts (ibid.) also recognised the potential criticism that because the practitioner is close to the research there may be issues of validity and bias. I have however been aware of the challenge of objectivity and have sought to ensure that my close involvement has not impeded my ability to be objective. Rigorous processes, particularly around handling of the data have been applied to ensure credibility and reliability.

As a practitioner I sought to bring about changes to the professional knowledge and practice of the teachers I engaged with. Lodico, Spalding and Voegtle (2010 p349) identified that it is not only important to ask *has* change occurred; but also to ask *how* it has occurred? What are the phenomena that have brought about the change? Through my research I sought to gain deep insight and understanding of the nature of the changes taking place, in relation to the development of teacher subject knowledge and the possible reasons for those changes.

Central to practitioner research is the notion of the reflective practitioner which is a key characteristic integral to my values and beliefs as a researcher. The combination of being involved in the research as a practitioner and stepping back and reflecting on the phenomena from the perspective of others, i.e. the teachers involved in my study, were key features of my context. The notion of the reflective practitioner as defined by Schön (1983) and others has been central to my practice for the past twenty five years and has been a key tool in developing and improving my practice at all stages of my career, from a trainee primary teacher, through to a classroom teacher, a senior manager, a headteacher, a provider of teacher professional development and a strategic leader in the development of the teaching of mathematics in primary schools in England. My passion and developed expertise in the teaching of mathematics have been supported through my studies with The Open University, which has encouraged, developed and enhanced my ability to reflect. This reflection I believe has been central to my learning and development. In learning mathematics, which I continue to do, I am aware not just of the mathematical processes I am engaged in, but reflect on the structure of the mathematics and how it connects and fits into a web of interconnecting ideas, I get excited when I see a new link or connection and gain fresh clarity or insight. Continued engagement in mathematics as a learner I believe has enhanced my teaching of the subject, not just in the development of subject knowledge; but through the process of learning I have gained insight into pedagogy. In my teaching I have developed a sensitivity and awareness of my actions in the moment, as discussed by Mason (2002). I notice

things that perhaps a novice teacher would not; in terms of the choices I make, the actions I take and the response and interaction I gain, from those I work with. I recall past events and interactions and bring them forward to improve present interactions, engaging in an iterative process of continual development. However I have recognised that the reflection when applied to research; as discussed by Cohen et al., (2007), required more careful, more systematic and more rigorous consideration. The data collected from the reflection of the research participants has in turn stimulated my own reflection within the context of practitioner research.

As with any research approach practitioner research is complex and diverse in its interpretation and application. My particular application has been that of an interpreter couched in the interpretation of my practice as seen by others and seeking to understand the phenomena that occurred in terms of teacher development. Significant to the process of interpretation has been the gathering of views from those on the receiving end of my practice, the teachers involved. The aim of the research process has been to understand and make improvements to my practice as a practitioner involved in the professional development of others.

Phenomenography

To understand what took place in relation to the development of teacher subject knowledge I have applied elements of a phenomenographical methodology to my research. The intention of my research has not been to evaluate the professional development programme which formed the context of my study but rather to understand the phenomena that occurred within the process of professional development in order to provide deeper insight both into my own practice and teacher development. An important aspect of seeking to understand the phenomena that developed was to gain insight from the participants' perspective, how they saw their developing subject knowledge and its relationship to their practice. This has provided me with significant insight, stimulated my thinking and contributed to the development of theory.

The concept of phenomenography as a research methodology emerged in the 1980s. It aims to understand phenomena from the perspective of others (Marton 1981) and has developed particularly within the context of higher education. Its aim is to understand how concepts and ideas are perceived within the context of teaching and learning (Tight 2015) and thus is particularly relevant within the context of my Master's level programme. There are several key characteristics which define the approach that I have applied to my study. Firstly it seeks to understand the phenomena from a second hand perspective, rather than firsthand, seeking to understand how the receiver perceives it, rather than what the phenomena actually is (Marton 1981). This is important within the context of my practice as a provider of professional development, it is crucial that I understand my practice from the perspective of those on the receiving end. The collection of data in the form of description is important within the context of phenomenography in order to capture the phenomena as seen by others. Data collection is usually through the medium of interviews but other data collection instruments can also be used. I have used both interviews and questionnaires.

Essence is another key characteristic of phenomenography. Ehrich (2003) described essence, as *the core meaning of an individual's experience of any given phenomena which makes it what it is* (p46). Within phenomenography essence is extracted from the variation in perceptions by individuals but is then categorised and treated collectively. Akerlind (2012) argues that the study of phenomena from individuals' perspectives within the context of phenomenography is sometimes misunderstood. The emphasis is not on the individuals' perspective but on the perspective of the group, categorising and bringing together ideas into an inclusive structure. My study has not only been experiences of individuals but the linking of common themes that emerged from a range of individuals.

Other key characteristics of phenomenography are seen by Marton (1981) within the treatment of the data. Data is categorised in order to identify the various ways in which the phenomena is experienced and perceived. This I have done through a coding process, which is discussed later in this chapter. Marton (1981) argues that there are a limited number of ways to experience and perceive phenomena and that by capturing these, the variation of the essence of the phenomena can be understood. A key development from my perspective is that the potential insight that this process develops can result in application and improvements to my own practice. This pedagogical potential is recognised by Marton (1981) as an important benefit.

At the initial stages of a phenomenological approach there is openness to what might emerge and a temporary bracketing of the researchers own ideas. However interpretation is required as ideas are brought together, analysed and categorised by the researcher. Akerlind (2012) recognises that the final product will inevitably reflect both the research participants and the researcher's interpretation of the data. Within the process are two levels of interpretation; firstly on the part of the teachers as their narratives are reported and their experiences described and secondly interpretation on my part as the researcher as I sought to understand and interpret their experiences. Teaching mathematics is a common everyday experience for teachers, something which they have lived and breathed for many years. Also engagement in the Northampton MaST programme became a common lived experience over the two years of the programme. My analysis was designed to enable me to understand through reflection what is significant within teachers' experiences and connects other experiences and events. I was particularly interested in teachers' reflection on experiences through the duration of the programme which had the potential to bring to mind past experiences in relation to both their teaching and personal experiences of learning mathematics. The process of bringing these experiences together, involving both contrast and synthesis seems to have been valuable in the process of interpretation on the part of the teachers and also assisted me in the process of seeking out an

accurate interpretation of their interpretation of the phenomena that took place within the context of the Northampton MaST programme. Issues of validity and measures to ensure that my interpretation is a valid one are discussed later in this chapter.

Phenomenography recognises (Marton 1981) that there will be variation in the interpretation by individuals of a shared experience. Ideas are never single entities which are transferable in a consistent manner, resulting in the same impact on individuals. Individuals may perceive things differently and their experience may be different from that intended. It has been necessary to recognise that the development, of subject knowledge had the potential to emerge in a different form than that envisaged or intended. Capturing exactly what did take place and the essences of the resultant phenomena was important.

Research Methods

Under the umbrella of practitioner research a variety of approaches to the collection of data can be applied (Lodico et al., 2010). I selected a mixed methods approach for the collection and analysis of data. Mixed methods research has developed as a third alternative to a purely quantitative or qualitative approach and seeks to combine the two. Quantitative and qualitative research paradigms have traditionally been distinct fields of research with researchers holding different, but equally strong philosophies. Those advocating quantitative methods often hold a positivist view of research where the search for objectivity is not only desirable but possible and data is viewed as context free and generalisable. Some educational researchers believe that this type of research might be applicable to the field of science from where it developed but less so to field of education due to its human, social and dynamic contexts. Thus an alternative approach to research has developed in the form of qualitative research which recognises contexts, particularly the social nature and influencing factors within those contexts. Qualitative research is often too small scale to be generalisable but theory is developed from the data often in the form of

Grounded Theory (Strauss and Corbin, 1997); which was applied to my research and is discussed later in this chapter.

A mixed methods approach reflected my pragmatic nature as a researcher. I was of the opinion that collecting a mixture of both quantitative and qualitative data was a practical solution to getting the most from my context and gaining both depth and breadth. Johnson and Onwuegbuzie (2004) provided an extensive list of the characteristics of pragmatists, including the desire to find the middle ground between extremes of philosophies. They also claimed that *a key feature of mixed methods research is its methodological pluralism or eclecticism, which frequently results in superior research* (p14). I hoped that this would prove to be the case, although I suspected I would lean more heavily towards the qualitative research approach, due the depth of insight it had the potential to deliver; which indeed I did. I sought to be flexible to adapt and develop the approach as data and findings emerged. Both, Cohen et al. (2007) and Lodico et al. (2010) claimed that within the field of practitioner research it is possible and indeed desirable to take a formative approach and direct the research along the most relevant and interesting paths as the data and findings emerged.

Quantitative research requires a large number of responses to be credible and this was possible within my context due to the large number of participants engaged in my programme. It also seemed to be desirable from an inclusion perspective to provide all with the opportunity to participate in the research and provide an overview across a large sample of the population. However I also recognised the limitations of quantitative research in getting beneath the surface and providing depth of insight into the nature of the impact and potential reasons for actions and outcomes. The main purpose of the quantitative data was not to apply a positivist approach to the research but instead support triangulation of the data where quantitative data might support

qualitative findings. Thus a combination of both quantitative and qualitative research seemed desirable.

Research Instruments

In order to gather both quantitative and qualitative data within the context of a mixed methods approach two research instruments were used. These were questionnaires and interviews.

Questionnaires

The construction of questionnaires is a complex process and achieving the desired outcome of acquiring data which answers or addresses the research questions and is reliable and credible is not an easy task. A number of factors have to be taken into consideration (Cohen et al., 2007). A key factor in the design of the questionnaire was the necessity to capture what the respondents wanted to say rather than bias the responses towards what the researcher wanted to hear.

Questions were devised for the questionnaire which had the potential for generating both quantitative and qualitative data. Some required a yes or no answer or selected response which could then be expressed as a simple percentage (questions 1 and 6a). Another (question 6b), although it allowed for an open response, appropriate responses were limited and allowed for those responses to be counted. Common themes that emerged from responses to other questions have also been counted (questions 4, 5 and 7). Most of the questions had an open element which allowed respondents to answer in their own way or provide an alternative response; to seek to counteract bias. Bailey (1994 cited in Cohen et al., 2007) recognised that open questions within questionnaires are appropriate if the possible answers are unknown and may also provide the opportunity to present rich and personal data. The questions relate to the development of mathematics subject knowledge and related practice and provided the opportunity for both quantitative and qualitative processing. I decided I could not, and did not want to, pre-empt the answers to the questions and thus their open nature was to allow participants to respond in their own way. I have however provided some scaffolding to question

four by inserting the phrase; *if making reference to course structures*; as I wanted to include data that would allow me to analysis the impact of the programme design, as discussed in the introduction to this thesis.

The open questions within the questionnaires were unlikely to capture the detail or reasons for changes to subject knowledge and practice to the same extent that I hoped my interview data would. They have however supported the triangulation of data and through the large number of responses have given some indication as to whether the findings from the interviews were likely to be representative.

My questionnaire was administered at the end of the two year programme for cohorts one, two and three. I analysed the questions; using the list of potential pitfalls provide by Cohen et al. (2007) to seek to ensure that there is clarity in terms of interpretation and provide for the range of responses that might be required. The complexity of getting the questions right to yield appropriate data is complex and I have had to make slight adjustments across the three phases of administration. Also not all of the questions presented in the questionnaire have been used for the purpose of the research. Those used are outlined in Appendix 2.

There are advantages and disadvantages in the use of questionnaires to gather research data (Cohen et al. 2007). One disadvantage can be the low response rate and return bias; whereby only those who are particularly in favour of the programme or those who have an axe to grind, make a response, which results in the sample being unrepresentative of the target group. However a sound response rate of 28% was achieved. All questionnaires in this research were administered electronically.

Validity and Reliability of the Questionnaire

It is argued that the benefits of questionnaires over interviews are their reliability (Cohen et al., 2007). They are anonymous so likely to encourage honesty. However the data within them may be unreliable due to a low response rate. I have collected a good representative sample of 28%, giving 196 completed questionnaires to support a claim to the data being reliable. The questions on the questionnaire were designed to be objective and to ask about the teachers' experience of the programme; thus they can be said to be valid as they were asking about the areas that the research was set up to enquire about. However I am aware that responses may be influenced by other factors. Some questions may be answered with what is considered the *correct* response rather than a *true* response. For example it is unlikely that teachers would indicate that the programme had no impact on their development, however this is also unlikely to be true, since the questionnaire was administered to participants at the end of the programme. Participation was voluntary and so for those who were not benefiting would probably not stay the full two years of the course. Also questions may be interpreted by different respondents in different ways and thus seeking clarity in the phrasing of the questions has been important. Reliability is also dependent on the way the questionnaires are handled and analysed. The qualitative data within the questionnaires is a particular challenge, requiring rigorous coding and analysis, avoiding a focus on data that supports my argument whilst ignoring other elements. This also applies to the interview data, which will be discussed in more detail below.

Within the reporting, analysis and discussion of the data the questionnaires are referenced as follows: Questionnaire/Cohort Number, followed by Question Number, followed by Line Number. The line number refers to the line of the spreadsheet where the data was entered. There is a separate line for each participant. For example: Q1 Q4 L55 refers to the first questionnaire, which was administered to cohort 1; question 4 of that questionnaire and Line 55 (the response from participant 55).

Timetable for Distribution of the Questionnaires

January 2012 Questionnaire to all cohort 1 teachers

July 2012 Questionnaire to all cohort 2 teachers

July 2013 Questionnaire to all cohort 3 teachers

Interviews

My second research instrument was that of interviews, specifically group interviews. Cohen et al. (2007) recognised the qualitative nature of data generated by interviews and saw it as a move away from seeing data as divorced from individuals and recognised the social construction of knowledge generated through conversation. The construction of knowledge aligns with the notion of *constructivist grounded theory* (Charmaz 2003), where within the context of the interview knowledge and interpretation are constructed by both the participants and the researcher.

The dynamics of group interviews are different to individual interviews in that within the individual interview the exchange is between the interviewer and interviewee, however within group interviews the exchange is also between participants and is therefore more diverse. There is the opportunity to build meaning through the exchange of dialogue between several individuals. Corbin and Strauss argued that *concepts and theories are constructed by researchers out of stories that are constructed by research participants who are trying to explain and make sense out of their experiences....both to the researcher and themselves* (Corbin and Strauss, 2014 p26). Cousin (2009) argued that the group interview is appealing and particularly relevant to research within higher education since it extends the academic practice of group discussion which is likely to be familiar to students within this context. I believe this proved to be the case within my context, particularly since the groups that come together already have an established mutual trust and understanding. The dynamics of group discussion allow for multiple contributions and

have the potential to enrich the discussion and subsequently the data gathered. They also provide opportunities for sharing and making comparisons. I believe it can also be argued that within group interviews there is greater number and variety in the potential lines of enquiry that emerge due to the presence of several individuals rather than just one.

The participants involved in my group interviews have had a common experience; however what they take and how they interpret the experience may be very different, this is a key consideration of phenomenography as discussed by Marton (1981). Drawing from a variety of sources has the potential to deepen and clarify my understanding of the participants' experience when taking part in the programme under study.

Since participants were self-selecting there was the potential to create bias within the group. However the data was analysed both within groups and also across the groups, allowing for comparison to be made. The facilities within the CAQDA software have allowed me to track whether an idea is limited to one group or spread across more than one group. Greater emphasis or credibility has been given to consistency of ideas across more than one group, thus supporting the validity of the findings.

McLafferty (2003) outlined the difficulties and limitations of group interviews and identified that group size is important since any more than six participants would potentially not allow all participants to contribute sufficiently. At his suggestion I decided to include between four and six participants within my small group interviews to allow all participants the opportunity to participate and still provide sufficient numbers for the potential of alternative views to emerge. I have also included whole group interviews of up to twenty teachers; however these were of a different nature where each person in turn was required to make one comment, thus ensuring

participation of each individual. These were useful in that they provided an overview of a relatively large sample of participants.

Boateng (2012) argued that group interviews may be more difficult to control than individual interviews in terms of the direction the conversation takes. This could be a negative aspect as participants may talk about ideas that have no bearing on my research questions. However on the other hand it could be argued that unexpected insight might be gained by the researcher having less control. He goes on to argue that the group may have undue influence on the thoughts and contributions of individuals within the group. Should a view be expressed by a dominant person in the group, a less confident member may not have the courage to speak up or disagree. This is an issue I have sought to avoid and have maintained an awareness of participants who either dominate the conversation or exhibit limited participation. In order to mitigate this situation I was aware of the potential need to intervene and bring others into the conversation. This only happened on a couple of occasions where a participant was not participating and I intervened and gave them the opportunity to speak. The challenge for me when leading the group interviews has been to focus and identify key ideas as they emerged. I have needed to ensure that these ideas were not lost within the dynamic interchange of dialogue, but fully explored. I did on two occasions intervene and bring the group back to a key focus.

Consideration of the role of the interviewer within group interviews is important. During my trial group interview I was struck by how much less I interceded, compared to individual interviews. Cousin (2009) argued that the role of the interviewer changes within a group context to that of a moderator where the role is to prompt and facilitate the discussion rather than control it. I found that one topic lead naturally to another and clarification was in the main facilitated by the group who sought to understand each other and build on that understanding through their contributions.

Cousin (2009) argued that the researcher needs to develop an awareness of group dynamics and context and how that might impact on the dialogue within group interviews. Her theory of interpretive repertoire I believe is particularly relevant to my context. *This concept is underpinned by the social constructionist position that the thinkable is constituted by the explanatory vocabulary and discourses available to us* (p57). The programme has developed a shared vocabulary in which to think about and discuss mathematics and this language was evident within the discussions. This had the potential to evidence teacher change and provide a means to express that change.

Questions were constructed to facilitate interviews of a semi structured nature. Some key questions were pre-defined and these can be found in Appendix 3. Each question was printed onto card and placed in the centre of the table at the appropriate time to introduce and maintain the focus of the question. The first question established the focus of the interviews and asks the teachers about change. The concept of change was central to my research questions; I sought to understand the changes that occurred in terms of the nature of the subject knowledge developed by teachers and also the factors which influenced those changes. Question two provided the opportunity to explore the impact of the five big ideas on teacher development, an element that at this stage was already found to be strong within the questionnaire data. Question three provided another opportunity to explore subject knowledge, the central theme within my study. Question four provided the opportunity to explore changes to pedagogy and it was anticipated that links would be made to subject knowledge, providing an opportunity to draw out the interplay between subject knowledge and pedagogy, thus addressing my third research question. I also had the opportunity to ask supplementary questions where answers to my research questions did not emerge within the conversations stimulated by the planned questions.

Within the reporting, analysis and discussion of the data the interviews are referenced as follows: Interview (Int) number as in the table above, followed by the time code. The time code is a reference to the segment in which the quotation can be found. For example: (Int1 25:36 - 25:55) refers to the first interview, where the quotation can be found between time code 25:36 and 25:55.

Timetable for the Interviews

Cohort 1 Autumn Term 2012

Cohort 2 Summer Term 2012

Cohort 3 Summer Term 2013

Validity and Reliability of the Interviews

The application of a common structure allowed for some consistency between the interviews and for comparisons to be made between common questions, thus supporting the reliability of emerging themes. However the fact that they are semi structured has meant that I have been able to move outside of the structure and have the freedom to intersperse unplanned questions in order to clarify information given and the interpretation I make of responses, again enhancing the reliability. There was also the opportunity to pursue emerging themes that arose within the interviews. Cohen et al. (2007) agree and claim that a significant advantage of interviews is that they can provide the opportunity for the interviewer to delve deeply into a topic through asking supplementary questions. They thus have the potential to provide a richer and more reliable source of data as they allow for greater interrogation of the information that is provided. Cohen et al. (2007) also identified that there are some disadvantages in collecting data through interviews. There is a substantial allocation of time needed to travel to particular locations and conduct the interviews and quiet and privacy were important considerations that I ensured for the interview process.

A key consideration in terms of reliability was the need to ensure objectivity and avoidance of bias on the part of the researcher. Kitwood (1997), cited in Cohen et al., (2007), argued that with skill bias can be largely eliminated. I audio recorded my interviews; and this has allowed me to scrutinise not only responses but how I conducted the interviews. I made use of a trial interview and through the process improved my technique to seek to ensure objectivity and credible data.

The personal and interpersonal nature of interviews requires that there is a relationship of mutual trust. This relationship had already been established, having worked with the participants for between one and two years; in the pursuit of shared goals, that of improving the teaching and learning of mathematics. The teachers in particular were used to voicing and sharing their reflections with me and the interviews were an extension of this process. This has supported both reliability and validity.

I have conducted eight group interviews spread across cohorts, one, two and three. The interviews were audio recorded and so all that was said was captured. The inclusion of my own voice has allowed me to analyse whether or not I have been over controlling and influence particular responses. A grounded theory approach (Glaser and Strauss 1967) was applied and this has allowed theory to develop from the data, supporting reliability.

Analysis

Both quantitative and qualitative analysis was applied to the research within the context of a mixed methods approach. The questionnaires, as outlined above were designed to capture data of both a quantitative and qualitative nature. The interviews were designed to focus on the gathering of data suitable for qualitative analysis and are also discussed above. Quantitative alongside qualitative analysis has added another dimension to the research and supported triangulation of the data. Quantitative analysis has allowed for responses to be counted to add strength to an argument developed from qualitative analysis. As the data was collected and

categorised I identified common themes from across the data and quantified their strength in terms of how often they occurred, for example the number of times *mathematical thinking* was mentioned by teachers in response to a particular question. This has provided a quantifiable dimension to qualitative data, indicating some level of commonality, which had the potential to add weighting to a theory.

Although I have adopted a mixed methods approach, qualitative analysis has remained central to my research. It has provided the mechanism for getting underneath the surface of the data to develop significant understanding. Merriam (1995) identified that qualitative research *is ideal for clarifying and understanding phenomena when operative variables cannot be identified ahead of time* (Merriam, 1995). Although there was a distinct structure and timetable to the programme, how the programme might develop and what might emerge were unknowns. Not all variables can be fixed at the start of the research; the research sits within the context of a learning process, where some elements might have been further developed, whilst others might have been dropped over the course of the three years of my involvement with the course. The phenomena was situated within the context of a professional development programme and I believed it was important to recognise that teacher development is not just situated within *training* sessions, but also within the context of the teachers' professional practice. A key aspect of professional development is not just how it impacts at the time, but how an initial input interacts with unknown or unpredicted variables and influences development over time. Qualitative research has allowed me the flexibility to dig deeper and identify the unexpected and develop fresh insights.

Questions of a qualitative nature have allowed respondents to make their own response, written or spoken in their own words and can be more difficult to analyse than quantitative data due to the need to interpret individual responses and place them into themes or categories. Cohen et al.

(2007) argued that *there is no one single or correct way to analyse and present qualitative data* (p461), but it should be analysed in a way that meets the purpose of the research. The purpose of my research was to understand the impact of features of my practice on the development of teacher subject knowledge and in turn its impact on their practice. It was analysed within the context of grounded theory, which is discussed later in this chapter.

Grounded Theory

A grounded theory (Glaser and Strauss 1967) approach has been central to my analysis of the data. It provided the means for an open approach that allowed theory to emerge from the data without prejudice. Grounded theory is a general methodology that can be applied to a number of research paradigms including practitioner research. Its aim is to *generate or discover theory* through or within the process of the research (Glaser and Strauss 1967). The process by which the theory comes about is through the systematic organization and analysis of qualitative data. CAQDAS software in the form of NVIVO 9 (QSR 2010) was employed to keep control of the large quantity of data generated by my study and assist in the coding and analysis process.

Grounded theory requires systemisation in the collection and sorting of data and a rigorous process with no preconceived ideas of what will emerge (Glaser and Strauss 1967). It can however allow for both critical and interpretive, creative analysis, however these ideas must remain grounded within the data (Corbin and Strauss 2008). I took the view of later proponents of grounded theorists that although an open approach is required, knowledge of the literature can enhance analysis, supporting sensitivity and openness to new insights providing it is evidence based (Giles et al., 2013).

The open approach required by grounded theory demands that the researcher remains open to new and emerging ideas. This necessitates concurrent collection and analysis of the data. This I

have done, particularly between questionnaires in an attempt to answer my research questions and gain deeper insight. For example between cohort one and subsequent cohorts alterations were made to Question 4 (see Appendix 2) to strengthen the emphasis on teachers own learning in mathematics rather than their pupils' development. I also added a question to strengthen the data I was gathering on teacher subject knowledge and asked teachers to name up to three key concepts or areas of mathematics where they felt their understanding has improved. There are several questions in the questionnaires, the data from which I am not using in this particular study; for example question 8 asked whether there were any significant factors outside of the programme that had impacted on the development of mathematics in teachers' schools. The data generated from this question was too broad and did not serve the purposes of understanding the impact of the programme on the development of teacher subject knowledge. The iterative process had focused my thinking and some of the questions became no longer directly relevant to my research.

I made limited changes to my interview questions. I constructed these questions after having collected some questionnaire data and engaged in some analysis. For the interview questions I have kept the basic stem questions the same and the semi structured approach, which is discussed below has allowed the flexibility to adapt and develop as my insight grew.

In grounded theory the collection of data is both theoretical and purposive (Corbin and Strauss 2008). The purpose of the collection of data was aimed at the generation of theory within the context of the research and reflected the research questions. My initial data was collected through questionnaires as discussed above and as I analysed the data I refined the interview questions to develop greater insight into the theory that was being generated. An initial analysis of pilot data collected through the questionnaires made significant reference to the five big ideas which formed the background to the programme, however *generalisation* was the least

represented. I wondered had this not had an impact, or was it just more difficult to analyse or identify? I therefore added a question to the interview structure to gain insight into this aspect and its impact on teacher development. The interviews were conducted at different times, which provided the opportunity to follow up on themes that emerged early, to develop greater insight and understanding.

Coding is central to grounded theory (Strauss and Corbin 2008) and is a process for sorting the data into manageable forms to support analysis for the purpose of generating theory. There exist different approaches to the sorting process. The categories can be defined beforehand and generated from hypotheses central to the focus of the research or allowed to grow from the language of the data (Schwandt, 2007). In the main I allowed the coded categories to grow from the language of the data. I believe this to be a more objective strategy which helped to ensure that nothing was missed. I have however allowed the pre-determined categories of the five big ideas (Mathematical Thinking, Representation, Pattern, Proportionality and Generalisation) as categories to commence with as I specifically wanted to include an analysis of the impact of this overarching structure of the programme on teacher development.

The coding process was conducted manually and methodically taking each section of data and creating categories in which to place segments. The facilities of CAQDA software facilitated this process. The starting point was an initial word search of the most frequent words used in questionnaire data (see Figures 1 and 2 below). In line with Corbin and Strauss (2008) this developed from a word association process to an interpretative process as the coding progressed. There was the movement from a mechanical to interpretive process, a movement from coding by words to coding by ideas as I gained insight and followed lines of emerging relevance. I subdivided or combined categories that did not use the same language, but appeared to be talking about the same thing; or alternatively used the same language whilst referring to different things. The

software allowed me to easily group, ungroup, and merge categories in order to make sense of the data. Wittingstein (1953) in his discussion of language and meaning talked about *seeing as* and I recognised the need to ensure that I was rigorous in ensuring that the data I viewed as *the same* actually was the same. Having established the categories they were labelled and these labels provided a useful language to talk about the data (Corbin and Strauss, 2008). Language is a tool to develop thinking and support not only external but also internal communication and generation of ideas and analysis of the data (Sfard, 2008).

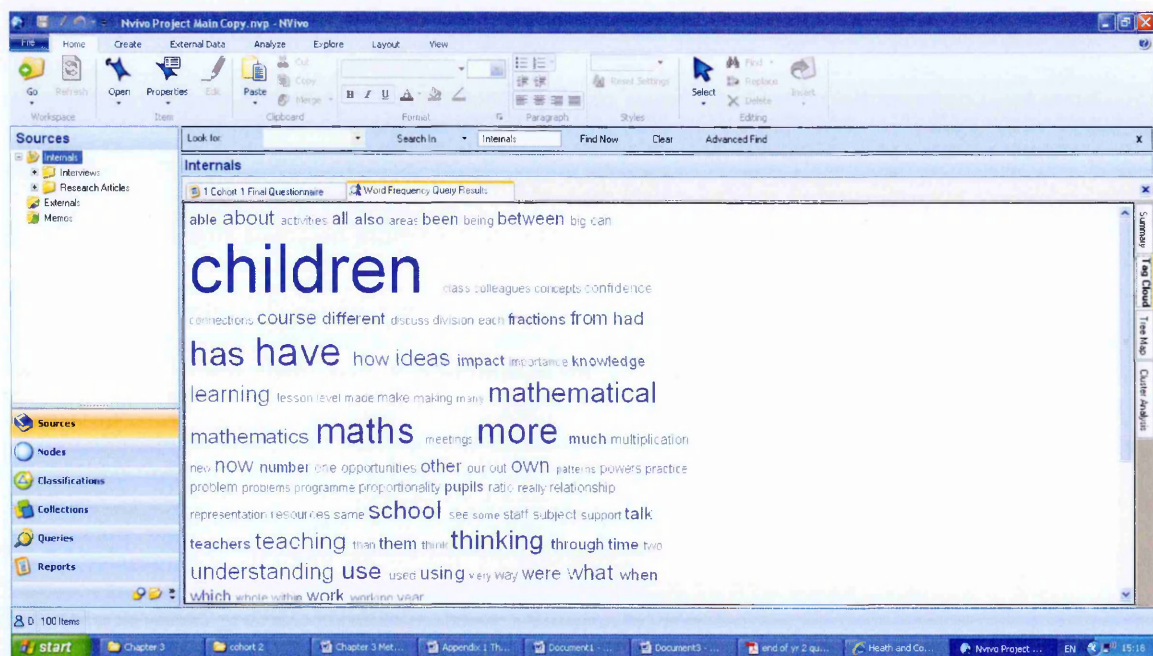


Figure 2 Tag Cloud Word Frequency Search in NVIVO 9 (QSR 2010)

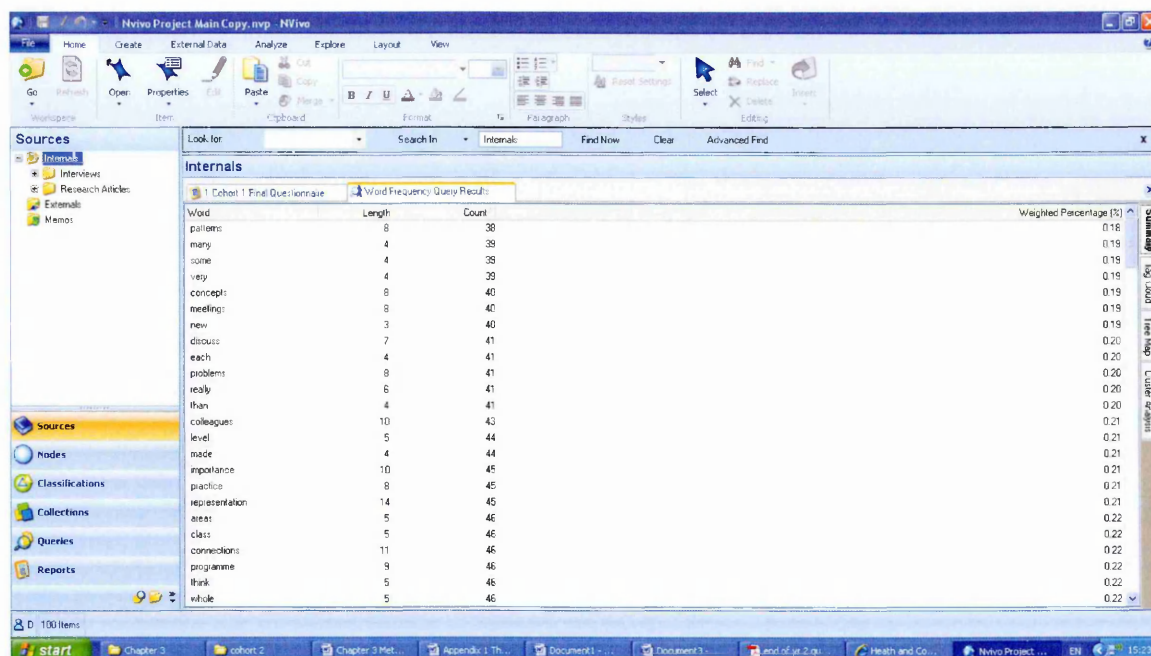


Figure 3 Summary Word Frequency Search in NVIVO 9 (QSR 2010)

Hutchinson (2010) argued that the development of codes should be representative of the data as a whole and cover a wide range of observations. This I have sought to do and in fact coded virtually all of the data but then focused in on areas that became relevant to my research questions. Sometimes sections of data, either from the questionnaires or interviews were placed in more than one coded category as they had multiple relevancies. An error of coding data to the same code twice is overcome by the software that will recognise this and only record it once; this keeps the number of references to a given category accurate. The codes were created for each set of data in turn, for example Questionnaire 1 or Interview 4 and then merged in order to bring relevant themes and ideas together. These codes were thus not in the main pre-determined but emerged from the data.

There were many advantages to the use of CAQDA software; it enabled flexible and easy searching and ease of identification and synthesising of the data. I particularly valued the feature of being able to easily return to the original source of the data, which then had the potential to clarify or provide a fresh perspective, and using human analysis and interpretation I was able to draw conclusions. This was particularly valuable within the context of audio data where I was able

to return and listen and return to the original source, listening to the nuances in the speech and the interaction surrounding it, placing it in context and aiding the analysis. The facility to rearrange data and present it in tables and other formats to support quantitative analysis was also helpful.

It has been argued (Corbin and Strauss, 2014) that there is not one reality but the potential for multiple interpretations. This reflects the idea that concepts and ideas are invented rather than discovered. In one sense I hold to this, particularly from the constructivist view point, that knowledge is constructed through interaction. However I believe that the process I engaged in sought not just any interpretation but a valid one, a valid insight into the phenomena that took place within the context of the Northampton MaST professional development programme. These interpretations were validated through the data provided by multiple individuals who developed a consensus of the phenomena that took place in terms of their own development and its impact on practice. I revisited the original data several times to ensure that I identified codes that captured all of the data. I then focused in on codes that were particularly relevant to my research, in that they addressed the research questions. I analysed and synthesised the information available.

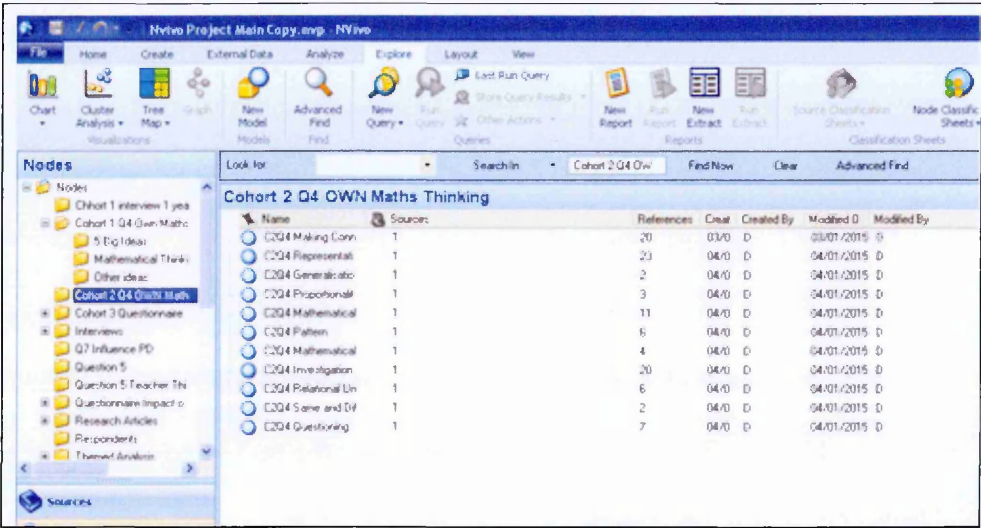


Figure 4 Creating Nodes (Codes)

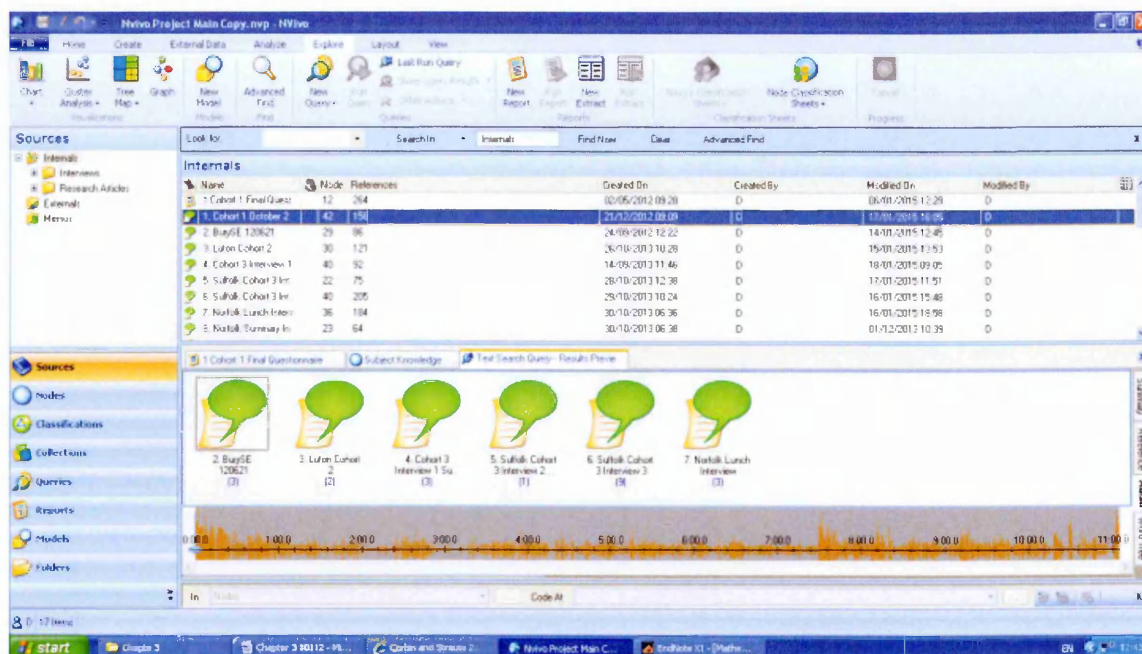


Figure 5 Coding Audio Files

The interview data was handled in a slightly different manner to that of the questionnaire data. The QAQDA software has the facility to code audio files without the need for transcription. I coded the data through listening rather than sorting text (Figure 3), capturing segments of talk and placing them into coded categories. I initially only partially transcribed audio data to provide an overall visual view of the data and fully transcribed where I saw the need to use direct quotations in my thesis. In the end most of the data was transcribed even if I did not directly quote from it. This enhanced the ability to search the data. I did however continue to use the audio facility to listen to the coded data in its original audio form, rather than its transposed written form. A sample of coding can be found in Appendix 4.

Coding is more than just making a list of labels to sort the data. Corbin and Strauss (2008) argued that it is about interacting in a meaningful way with the data. Making comparisons is important within the process. Hutchinson (2010) argued for the need to make comparisons across the data at every stage. Are common ideas emerging or are there contradictions in the data and can these

be reconciled? I had a huge amount and range of data and sought to make comparisons, looking for both contradictions and commonalities. The facilities of CAQDAS assisted me in making comparisons through the ease of movement of data.

Grounded Theory and theoretical density in the advancement of theory

Theoretical density involves saturation of the data and this is important within the context of grounded theory. It is achieved when all of the observations have been presented, nothing new can be found and nothing has been missed. Corbin and Strauss (2008) argued that this process supports the generation of theory. They also argued that *saturation is more than a matter of no new data. It also denotes the development of categories in terms of their properties and dimensions, including variation, and if theory building, the delineating of relationships between concepts.* (Corbin and Strauss p 143) I sought to do this in my visiting and revisiting of the data. As theories emerged I returned to the data in order to secure my hypothesis. Through using a mixed methods approach and having a large amount of data gathered through questionnaires, I have been able to cross reference ideas that emerged from the interviews across a larger population provided by the questionnaires.

Issues of validity and reliability of quantitative and qualitative analysis

Handling and presenting quantitative analysis in a manner that is transparent and reliable is relatively uncomplicated compared to qualitative data. However there are key ideas that I have needed to take into consideration. Quantitative analysis is counted data, however it is easy to obscure the truth in how the data is presented. A percentage for example in the form of 60% may represent 6 out a total of 10 responses or 60 out of 100 responses. To address this I have sought to ensure that I am transparent in my reporting of quantitative data.

Ensuring validity and reliability of qualitative data is more complex and has required greater consideration. The nature of validity and reliability of qualitative research has been the focus of

much debate (Morse et al., 2008, Graneheim and Lundman, 2004, Merriam, 1995, Goetz and LeCompte, 1984) Two approaches have been identified, one is to apply the same standards to qualitative research as are applied to quantitative research and demonstrate how the research meets those standards (Goetz and LeCompte, 1984). Some, for example Morse et al (2008) argued that this is inappropriate as qualitative research is different both in nature and purpose and should have its own set of criteria to measure validity and reliability. For example the replicable criteria that is applied to quantitative research is inappropriate since with most qualitative studies it is impossible to reproduce the exact set of circumstances to test this criteria. An alternative approach then is to acknowledge qualitative research as being different in nature and purpose and to measure it against its own set of standards. For example Agar (1986) identified three measures in the form of, credibility, accuracy of representation and authority of the writer. The first that of credibility goes beyond just answering the question *is the research believable or plausible?* It also gives consideration to the match between the original data and its interpretation. This requires rigorous analysis on the part of the researcher, returning many times to the data, and asking *is this the only plausible interpretation of the data, or could there be another?* This I have sought to do, returning to the data many times. The facilities within the CAQAD software in the form of NVIVO (QSR, 2010), as discussed above has enabled me to quickly access and check the original data, including audio sections from the original interviews. It is argued that the use of CAQDAS is not only a research tool, but also impacts on the research philosophy and approach. Some purist users of qualitative research would not advocate its use, claiming that it lacks the robustness and rigour required of qualitative research (Baugh et al., 2010). Having used it, I can see the merits of this argument if using just the automated facilities and I acknowledge that this might result in superficial analysis and an ignorance of the context. However I have sought to *avoid the trap of letting the software control the data* (Catterall and Maclaran, 1998 cited in, Baugh et al., 2010). I have recognised the need to engage and make human and professional judgements in order to gain depth and make sense of the data.

Agar's second criterion is that of accuracy of representation. I have adopted this within the context of my research and believe it supports credibility. There is a process to go through from collection of the raw data to sorting and categorization, to interpretation and presentation of the findings, each of these stages have been handled with care and fidelity to ensure that the findings are a true representation of the original data.

Agar's (1986) final criteria, authority of the writer is not sufficiently robust and I would not feel comfortable being judged on this criteria, I have engaged in insufficient research and writing for this to be applied. However, I have been handed authority by the teachers on the course, to present their voices and views and I have done so as faithfully as I can. The act of analysis, knowing that it is systematic, careful, thorough and critical lends authority to my writing and to my conclusions.

Guba and Lincoln (1981) also suggested three criteria. The first was credibility as suggested by Agar (1986) and the second and third were dependability and transferability. Dependability I believe is linked to accuracy of representation as discussed above and the same measures are applied. Transferability is linked to generalisation, a topic much discussed in relation to qualitative research. Some such as Erlandson (1993) claim qualitative research is not transferable due to the fact that it sits in specific contexts with their own unique characteristics. However I do believe there are lessons learnt and principles developed that can be applied to my practice and potentially that of others.

A key issue that requires consideration within the context of all qualitative research is that of interpretation. Qualitative research inevitably requires interpretation and often, as in my case interpretation of the views of others. Within this context, the question might be asked; can the process ever really identify the truth? I believe that I have identified useful realities that can be

applied to my future practice. These realities might not be comprehensive, in that I might not know or understand everything, but what is learnt is valuable and applicable to my future practice.

Validity and reliability can only be achieved when relevant criteria have been identified and applied with rigour and consistency. The criteria I deem most relevant to my research to ensure validity and reliability are:

Rigour has been applied to handling of the data in terms of sorting, categorising, synthesising and abstraction of meaning from the data. The process I went through to achieve this was to systematically work through the data in the coding process, checking that nothing had been missed and constantly asked if there were alternative interpretations. I ensured data could easily be returned to its original context, for example in the midst of the discussion that went before and after within the context of an interview in order to clarify and check meaning and ensure that interpretation was correct.

Sensitivity and responsiveness are relevant and important. Morse et al. (2008) identified lack of responsiveness to be the greatest threat to the validity of research. I have applied their requirement for the researcher to *listen* to the data, sort, re-sort and analyse until clarity is achieved, abstracting what is there, and not what I thought was there or would have liked to have been there. I have been through the data many times and in the case of the interview data actually listening, picking up on nuances and surrounding data to interpret was actually was being said.

Verification is a process of checking that the process of data handling is secure in order to lead to credible findings. There are various techniques I have used to support this process. This has included triangulation of the data, checking that what was found in one interview was consistent with other interviews and the large sample of questionnaires. Being able to track the source of each element of data and then looking at the spread of ideas across multiple sources was

important in supporting verification and adding strength to an argument. When I have been unclear during the interview process of what was being said I asked for clarification to check that my interpretation was correct. The process of grounded theory as discussed later involved verification through its match to the data; all arguments have been grounded in the data.

A useful question asked by researchers in relation to reliability is *would the same key themes have emerged if someone else were to sort and classify the data?* I believe that similar themes would have emerged as they are strong within the data. Indeed I did trial this and asked an independent researcher to carry out some sample coding of the data and very similar themes did emerge and there were no significant differences. However interpretation and the development of theory may well have been different, in that alternative lines of enquiry may have been pursued. However key ideas emerged strongly from the data, ideas that could not have been overlooked. Issues of bias have also been considered. Rigorous handling of the data has been applied; ensuring that all data was coded and checked to ensure nothing has been missed and I constantly asked if my interpretation was the most likely among any alternative interpretations.

Triangulation

Creswell (2012) describes triangulation as *the process of corroborating evidence from different individuals, types of data, or methods of data collection* (p259). My research study collected data from teachers engaged in the programme, both a small focused sample through the interviews and a wider sample through the questionnaires. The data was in the main of a qualitative nature with some quantitative and collected through the medium of questionnaires and interviews. The variety of data selected and collected was designed to capture and address the research questions and support the reliability and credibility of the findings. The wider sample represented in the questionnaire data supported triangulation of the smaller sample of data represented in the interviews. Being able to track data as discrete cohorts enabled comparisons between

cohorts, this provided for another form of triangulation; where outcomes are repeated across more than one cohort of teachers triangulated and strengthened the findings. Common themes were also grouped across the coded data. The CAQDA software allowed for identification of each source of data where multiple sources were brought together. It also counted the number of references. This enabled me to look at the range of data that included a common theme. Where there were multiple sources from both the questionnaire and interview data this added strength to the findings. This is reported on in the Chapter 4 of the thesis.

Ethical Considerations

There are various ethical considerations with regard to the above research, involving respect for the individual, anonymity of participants and careful and accurate handling and reporting of the data. There are particular issues regarding practitioner research and these are discussed within practitioner research methodology literature, (for example Cochran-Smith, M., & Lytle, S. L. (2009)). A key issue relates to my relationship with the participants on two levels, one of an authoritarian position and the other of familiarity and established relationship. The first had the potential to involve issues of coercion where participants feel that they should engage in the research process in order to get good grades or provide a positive opinion of themselves as keen and reflective students. The second may have involved teachers feeling they ought to engage in order to help me as a person they have a positive relationship with. I have sought to guard against these two issues by ensuring that participation in the research was voluntary. The questionnaires were anonymous, so I would not know whether or not a teacher had participated. I only needed a small number of teachers for the interviews and so the majority did not take part, ensuring that those who did not engage were not singled out.

Within the process of the interviews it may potentially have been the case that teachers provide the answers they think I want to hear rather than the reality. The interview data however was

triangulated with the questionnaire data, where due to anonymity the answers are more likely to be honest ones.

An ethical code to address these issues has been established and approved by the University of Northampton's ethics committee (Appendix 5). The Code is informed by the principles established in the Revised Ethical Guidelines for Educational Research (2004) issued by the British Educational Research Association (BERA). The collection of data for the research in the form of questionnaires and interviews is outside the normal remit of course evaluation and so required express permission. Questionnaires were sent out electronically and participants have the option of not completing. The email contains the ethical code and participants are informed that by completing they are giving their consent to participate in the research. Participants are asked within the questionnaire for consent to be contacted for interview and are informed of their right to withdraw at any point.

Special ethical consideration regarding the use of recordings has been given to the interview process. Prior to the interviews participants are asked for permission to record the interviews and are given the right to refuse recording. Should this occur I will ask for permission to take notes. Cohen et al. (2007) points out, it is more difficult to make interview data anonymous; however identification and names were removed at the point of entry into Nvivo 9 (QSR 2010), prior to the analysis. Original recordings were stored in a locked filing cabinet and destroyed at the end of the project. A signed declaration of permission to use the recording was obtained from each participant.

Summary of Research Methodology and Methods

In summary I have conducted my research study drawing on elements from different methodologies, but bringing these into a coherent and rigorous process to enable me to develop

greater insight and understanding of the nature of teacher subject knowledge, how it developed and its influence on practice. I began the research from the standpoint of a practitioner, seeking to investigate my own practice as a facilitator of teacher professional development. I have sought to uncover and understand the phenomena that occurred within a particular professional development programme which took place within the context of my practice. I adopted a phenomenological approach in the Heidegger (1970) cited in (Ehrich, 2003) interpretation where the researcher is not distanced from the research, but closely involved, bringing their own reflection to bear on the interpretation of meaning. Data was collected and analysed using a mixed methods approach involving both quantitative and qualitative analysis. Quantitative analysis was applied to some of the data to provide an overview and insight into the strength of emerging ideas from across the data, for example, how many times did participants make reference to the use of multiple representations in supporting learning of mathematics? It also provides triangulation where findings from within the interview data were verified from the larger sample contained in the questionnaire data. Qualitative analysis was applied to the data gathered through the interviews and the open questions within the questionnaires to uncover the nature of teachers' development, the influencing factors and its interaction with practice. A grounded theory approach was applied to the handling of the data, in the form of a coding process where insights, phenomena and theory emerged from the data. This approach allowed for theory to emerge from the data, but not in a completely *empty mind* mentality as originally advocated by Glaser (Heath and Cowley, 2004), but in an interpretative one (Corbin and Strauss 2008).

The analysis process has drawn on data, extracted from a range of questions asked through the questionnaires and interviews. The primary purpose of the questions used to gather data in the research were to facilitate teachers engagement in self reflect and talk about their experiences, rather than necessarily answering the specific question asked. In answering any one of the

research questions, data was gathered from across the spectrum of data in seeking to understand the phenomena that occurred.

Within phenomenography meaning is a co-creation of the researcher and the researched, the grounded theory approach I have adopted complements this approach. As stated previously I have not adopted a completely empty minded approach. I recognised that there are potentially multiple interpretations of the data (Mishler, 1979), however my interpretation has been informed by my own reflection and analysis, but remains grounded in and supported by the data. It is an interpretation that seeks to understand the phenomena that took place within the context of my professional practice. The influence of this has shaped the theory that emerged (Mishler, 1979); this is valuable and relevant to the development of my professional practice. I support the recommendation of Heath and Cowley (2004) that *It is wise to remember that that the aim is not to discover **the** theory, but **a** theory that aids understanding and action in the area under investigation* (page 149). My aim has been that the study should improve my practice and the actions that I take in the future.

Chapter 4: The Findings

The study seeks to understand the nature and development of teacher subject knowledge and its interrelationship with and influence on professional practice within the context of the Northampton MaST Programme. To facilitate this aim data has been gathered from three cohorts of teachers engaged in the programme, through the medium of questionnaires and interviews as discussed in Chapter 3. The data has been sorted and categorised using CADA software and the findings that have emerged are presented in this chapter. This will include consideration of triangulation of findings. As discussed in Chapter 3 two key forms of triangulation are included in the study. The first is the match between the findings in the interview data and the questionnaire data, findings which emerge from both will strengthen the arguments made. Secondly the commonality of findings between three cohorts of teachers will suggest that similar phenomena have occurred and support the trustworthiness of the conclusions.

This chapter will:

- Present the findings from the questionnaire data
- Present the findings from the interview data
- Discuss how the findings answer the research questions

Findings from the questionnaire data

As indicated in chapter 3 responses from only a selection of the questions from the questionnaire were used for the purposes of the research. These are summarised in the table below and outlined in full in Appendix 2.

Question	Focus
1	Meeting expectations in terms of development of subject knowledge

4	Impact on mathematics
5	Impact on practice
6	Identified areas of developed subject knowledge
7	The influence of the programme structure, content and organisation on teacher development

Table 1 Questionnaire Questions

Minor adjustments were made to the wording of the questions between cohorts (see Appendix 2). The return rate is indicated in the table below:

Cohort	Number of questionnaires returned	Percentage of questionnaires returned
1	108	42%
2	46	21%
3	42	21%

Table 2 Questionnaire Return Rate

Meeting expectations in terms of development of teacher subject knowledge

This question asked: *In what ways has the programme met your expectations in terms of subject knowledge.* The data gathered was exclusively quantitative in nature and is presented in the table below showing the responses for each of the three options offered:

Not Met	1%
Met	30%
Exceeded	69%

Impact on mathematics

This question asked: *What elements of the programme, if any have had a significant impact on your OWN thinking and mathematical development? Please indicate how your thinking has changed or developed. Please name up to three aspects. If making reference to course ideas, please be specific.*

The key themes that emerged are presented in the graph below:

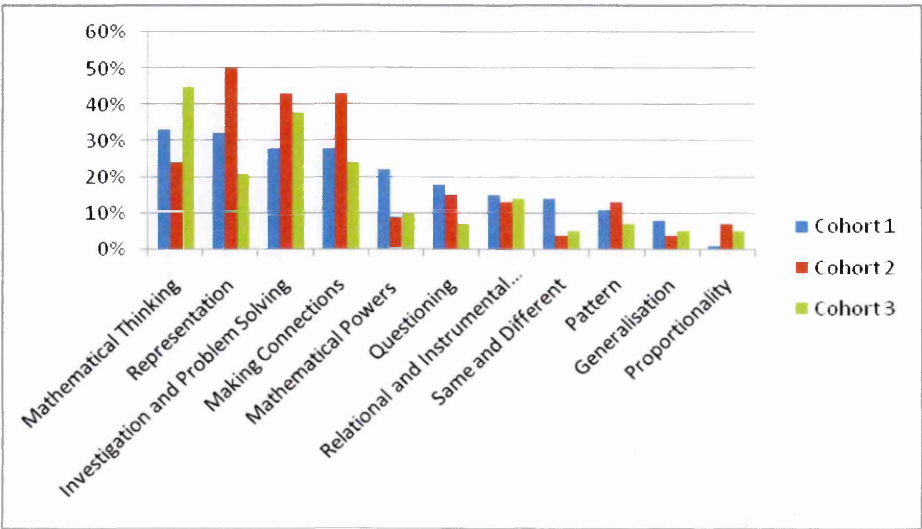


Figure 6 Question 4 Mathematical Development

The question was asked of all three cohorts and is open in nature with no predetermined suggestions. It is interesting that a similar pattern emerged across all three cohorts with mathematical thinking; representation; investigation and problem solving; and making connections being the most common themes for all three cohorts.

Impact on Practice

This question asked: *What elements of the programme, if any, have had significant impact on your practice? Please indicate how they have impacted on your practice and consider how your*

practice has changed. Please name up to three aspects. If making reference to course structure, please be specific. The key themes that emerged are presented in the graph below:

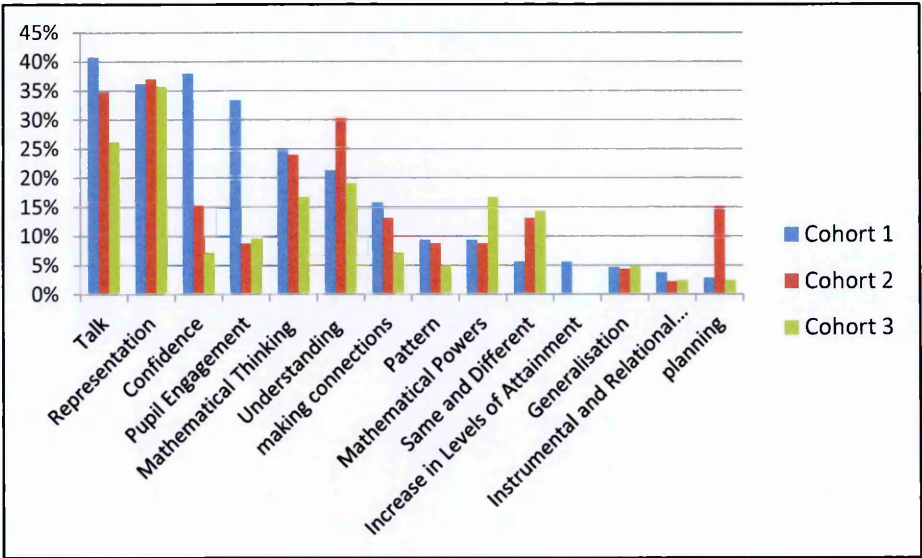


Figure 7: Question 5 Impact on Teachers’ Practice

There is a similar profile across all three cohorts of teachers with talk and representation being the strongest two themes emerging from the data. There are differences however in some categories. For example with reference to pupil engagement it may not have been mentioned by a teacher but that may not indicate that the teachers’ pupils were disengaged. In fact other references may suggest that they were engaged, in particular in talk and exploration and using practical resources.

Identified areas of developed subject knowledge

This question asked: *Has your subject knowledge of mathematics developed?*

Yes/No? Please name up to 3 key concepts or areas of mathematics where you feel your understanding has improved. This question was a later addition to the questionnaire, as discussed in Chapter 3 and was asked of cohorts two and three. 99.5% of respondents indicated that their

subject knowledge had developed and the graph below indicates the identified areas of development:

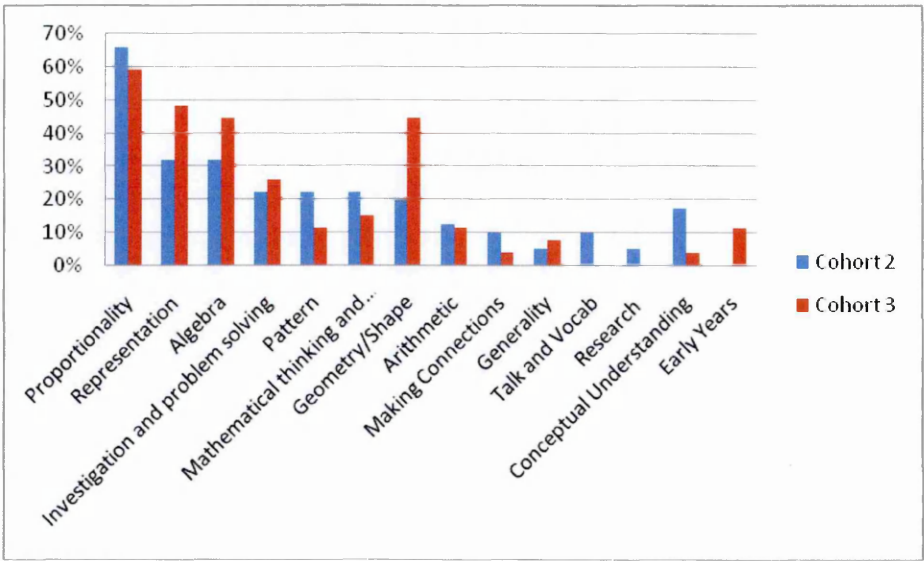


Figure 8 Question 6 Areas of mathematical development

As can be seen from the graph there is significant commonality between the two cohorts in terms of the identified as areas of development. I had expected that teachers might just list topics from the primary curriculum, such as algebra and geometry; they have included these, but other things have also emerged strongly. For both cohorts the top three are proportionality, representation and algebra. The strength of proportionality is perhaps to be expected as fractions, decimals, percentages and ratio, as evidenced in Chapter 2 are seen as the most challenging areas within the primary curriculum and algebra at the time was not formally part of the curriculum, so would be a new focus for many. What is more surprising is the strength of representation across both cohorts, with over 30% of teachers from the cohort 2 data and almost 50% from cohort 3 data identifying it as a key developed area.

The influence of the programme structure, content and organisation on teacher development

This question asked: *What key factors in terms of the structure, the content, or the organisation of programme do you think have influenced the impact on your thinking and practice as described*

above? A variety of factors were referenced to, and those referenced to the most and across all three cohorts are indicated in the table below:

Name of Category (Node)	Description of Category	Number of References
Networking	Where reference is made to the value of interaction with other professionals	38
Academic Study	Where reference is made to engagement with research literature or other academic study	33
Big ideas	Where reference is made to one or more of the 5 big ideas	32
Pedagogic Constructs	Where reference is made to the pedagogic constructs that the programme focused on.	28
Subject knowledge	Where reference is made to the development of subject knowledge, including learning mathematics in an experiential context	20

Table 1 Question 7 Factors that influenced teacher development

Findings from the interview data

The data presented in this section of the chapter are taken from the interviews conducted with all three cohorts of teachers as outlined in the table below. The questions asked in the interviews described as *small group* can be found in Appendix 2. The interviews described as *whole group* were asked just one question *what has been the impact of the programme on your development?* Each person was required to identify and name one aspect. The small group interviews followed the process of normal conversational interaction in response to the questions asked. The timetable for these interviews is outlined in the table.

Interview	Date	Cohort	Duration		
1	Nov 2012	1	42 mins	Small Group	1 year after completion
2	July 2012	2	12 mins	Small Group	On Completion
3	July 2012	2	18 mins	Small Group	On Completion
4	July 2013	3	22 mins	Small Group	On Completion
5	July 2013	3	7 mins	Whole Group	On Completion
6	July 2013	3	30 mins	Small Group	On Completion
7	July 2013	3	33 mins	Small Group	On Completion
8	July 2013	3	7 mins	Whole Group	On Completion

Table 2 Summary of interviews

The interviews were coded and a significant number of themes emerged from the data. These are recorded in the table in Appendix 6. Although the data is represented in a quantitative format this

does give some indication as to the strength of ideas within the data. However the data collected through the interviews was collected for qualitative purposes to provide insight into the development of teachers engaged in the Northampton MaST programme and to answer the research questions.

Answering the research questions

The data collected and the subsequent findings, was designed to answer a specific set of research questions. They are:

1. *What is the nature of subject knowledge developed by teachers engaged in the professional development programme?*
2. *How does teacher knowledge develop and what factors influence its development?*
3. *In what ways does the development of subject knowledge interact with and influence the practice of teachers?*

Particular questions were designed to answer each of these research questions as discussed in Chapter 3. However information has emerged from across the data for each of them and I have not limited the focus to any particular set of data in answering the questions. All information and insights from across the data will be taken into account in answering each of the questions.

Question 1: What is the nature of subject knowledge developed by teachers engaged in the professional development programme?

The data indicates that not only have teachers increased their subject knowledge, as would be expected within a programme focused on the development of subject knowledge, but there are also indications that the nature of such knowledge has taken on particular characteristics. This is indicated by the way such knowledge is described by teachers within the data. Teachers talk about relational knowledge, making reference to the work of Skemp (1976). They also talk about making connections, indicating a connective form of knowledge. The adjective deep is used a number of times to describe their subject knowledge. The big ideas of pattern, representation,

generality and mathematical thinking are also discussed in relation to subject knowledge. Each of these emerged from the data as categories (nodes) within the coding process. The findings from each of these categories are discussed in more detail below in relation to answering this first research question.

Relational Knowledge

Relational knowledge is referenced directly many times both within the questionnaire and interview data, with 26 references across the eight interviews and 34 references across the questionnaires. Often relational knowledge is referenced by the teachers to Skemp (1976), one of the early required readings within the programme. A teacher reports that *the Skemp article on instrumental and relational understanding in mathematics has really changed the way in which I think about mathematics* (Q1 Q4 L55). The data suggests that this was a new way of thinking for many of the teachers and has made a significant contribution to defining the nature of their developing subject knowledge. A teacher reflects that *there were things that we did that I had learnt at school by rote.....what changed in my subject knowledge was knowing why that worked* (Int1 18:59.7 - 19:18.4). Skemp's argument for the importance of relational understanding, knowing not only what method worked but why. Skemp (1976) became significant in terms of the development of teachers subject knowledge.

Teachers across the data equate relational knowledge with an understanding of *why* and talk about *really understanding why something works rather than just following a strategy* (Q1 Q5 L45). They contrast between previous instrumental understanding and new relational understanding *it made me look again and think ah gosh that's why that works, why had I not noticed that before?* (Int1 19:18 - 19:40) They see this form of knowledge as necessary for effective teaching of mathematics. *I am more aware of how I can help children to learn and develop their mathematical understanding rather than teaching a method with no*

*understanding.....I want children to know how a method works and be able to explain how they worked out the answer (Q1 Q4 L55). Teachers identified that this new relational knowledge was important to develop through their teaching. Before the course I never considered the difference between relational and instrumental understanding and the impact on children's learning. I now feel it is vitally important for children to have a relational understanding and really understand the 'how' and 'why'. (Q2 Q4 L110). The evidence from across the data suggests that teachers now see this form of knowledge as necessary for effective teaching and learning of mathematics, for example a teacher talking about change and influencing factors on her development reports *going on to understanding rather than the procedural and so far me it was Skemp's article and reading about relational ideas, relational learning, when I realised that all my learning ... had been instrumental ... that was influencing how I was teaching children, it was all about procedures, doing this, doing that and not actually making connections and thinking and understanding mathematically* (Int7 1:22.4 - 1:57.0)*

Connective Knowledge

The importance of making connections is strong both within the questionnaire data (71 references) and the interview data (31 references). The link is made with relational knowledge, making *structural/conceptual* connections in order to understand *why* a technique or procedure works. A teacher reflects that much of her past learning was instrumental and this was influencing how she was teaching, *it was all about procedures doing this, doing that, not actually making connections* (Int7 2:21.7 - 3:17.3). Teachers relate making connections to relational knowledge and the two are seen together, making connections in order to understand more deeply and understand why the mathematics works.

Connective knowledge is related to mathematics at the macro level, enabling teachers to see *the bigger picture of the maths..... and seeing how all the different areas are connected that helps at*

looking at all the relationships in maths.....I don't think I had that idea of the bigger picture of maths before this course(Int2 1:15.7 - 1:29.7). It is also related to depth the depth to which we go, looking at concepts and you are linking it all together (Int2 0:23.5 - 1:15.8)

Making connections impacts on the way teachers view mathematics, *my thinking has developed in the way that I see mathematics as a whole. Before the programme, I tended to think about maths as lots of different and separate aspects e.g. calculation, fractions, shape etc. Following the programme, I think more about maths as a whole, and particularly where connections can be made between different concepts in mathematics. My overall thinking in maths is no longer as fragmented as it was before.*

Deep Knowledge

The use of the adjective “deep” to describe the nature of the subject knowledge that is developing is significant, particularly in the interview data where it appears 22 times. *I know that's the very first thing we talked about when we started the course, it's about depth of understanding, and that's just become clearer and clearer (Int 3 0:49.0 - 1:20.1)* It's also used within the context of references to relational knowledge and/or making connections.

Pattern and Generality

Teachers were already familiar with the idea of pattern and there is mention of it across the data with 50 references. However for some the idea of generality was new, *no I'd never thought about it (before the programme) (Int7 13:24.4 - 13:26.2) and I'd never heard of generality before I did MaST (Int6 20:58.5 - 21:44.8).* The teacher goes on to link generality with pattern, *it's become a day to day routine in my class the children in my class enjoy generalising, they like to look for the pattern, specialise and generalise (Int6 20:58.5 - 21:44.8).* The link between pattern and generality is also made by a teacher who says, *it's seeing the patterns and making those general rules about the patterns,* thus recognising that generality is a step on from recognising a pattern.

Within the questionnaire data pattern is stronger than generality, however in the interview data generality was deliberately drawn out to interrogate teachers understanding and application of the concept. I was concerned that it was an area that featured so little within the questionnaire data that I decided to seek it out within the interviews. The references indicate that teachers are thinking about generality in their teaching, a teacher comments that *generalisation, before I wouldn't have used that and now.....we have become a class of everybody generalising whether it's the higher abilities or the lower abilities* (Int6 2:50.9 - 3:22.7).

Although there is limited reference within the questionnaire data to generalisation, the references that are there suggest influence on teacher development and their practice. *Generalisation has had a huge impact on my maths as I was taught instrumentally and I didn't always understand what I was doing- connecting ideas together and expressing a rule/generalisation has helped me enormously* (Q1 Q4 L20). Teachers are recognising links between generalisation, making connections and relational as opposed to instrumental understanding.

Representational Knowledge

The category of representation has emerged as one of the strongest categories within the five big ideas, with 38 references across the interview data and 140 references across the questionnaire data. Within the questionnaire, references to representation are equally strong across both the question on teachers' own mathematical development (Question 4) and their practice (Question 5). A definition of representation in this context of the data is adapted from Lesh et al (1987) to be any manipulable model, picture, diagram, or visual image that supports exposure to the structure and relationships within the mathematics. Different aspects regarding the use and purpose of representations come through in the data.

The data indicates a range of media to represent mathematics is being used. The most referenced to in the data are concrete resources, those that can be handled and manipulated (47 references). Also visual images are referenced to (25 references), diagrammatic or structured (13 references) and drawings (6 references) (See Appendix 4)

The need for a range of representations (multiple representations) is mentioned with the justification that one way of representing the idea may not work for all pupils (Int3 15:16.5 - 15:30.7) Reference is also made to representations aiding a learners ability to *see mathematics in different ways* (Int3 5:54.2 - 6:12.5). The representations' purpose is connected to the use of practical apparatus.....to develop children's thinking and learning (Q1 Q4 L4). They are being used to represent mathematics the use of practical equipmentto represent mathematical ideas (Q1 Q4 L4)

There is an indication in the data that alongside increased use there is a shift in terms of the nature of the usage of representations. For example a teacher (Int6 22:57.2 – 23:13.5) explains they are used in more sophisticated ways as children's understanding of concepts progress, indicating that they are not just used at the initial stages of learning mathematics. Also there is explicit reference to them being used with older children to aid understanding and that this is a new initiative (Q2 Q5 L55). An interesting point is made where a teacher identifies that representations are *vital for initial understanding, for developing deeper understanding, and for children to be able to develop beyond what the teacher teaches*. (Q1 Q4 L63). There is an indication of pupil independence in this quotation, which is also reflected in a number of references to children selecting, or developing their own ways of representing mathematics.

There is direct reference to how representations are being used to support the development of teachers own subject knowledge *representations, they've made it really clear and the amount of times I've sat here and gone Oh Yeh* (Int6 5:21.7 - 5:46.4). There is also reference to teachers making the right choice, relevant to the concept. The teachers need to *be able to select the representations carefully for that mathematical area* (Int3 14:45.6 - 15:16.5).

Mathematical Thinking

Reference to mathematical thinking is strong within the data, with 164 references across the questionnaires, and 26 across and included in all 8 interviews. Its impact is discussed as being both on the development of teachers' subject knowledge and their practice. The nature of what teachers' term mathematical thinking is almost always linked to the pursuit of making connections and the development of relational understanding as discussed above and these three ideas are often discussed together within the data. In terms of the impact on teachers' practice mathematical thinking is often linked in the data with increases in the use of mathematical investigation and problem solving opportunities within teaching and the use of the mathematical powers as outlined in Chapter 1, in particular conjecturing and convincing which has 40 references across the data.

You don't know what you don't know

To conclude this section on the development of teacher subject knowledge, an interesting observation arises within the interviews, that teachers prior to the programme were not fully aware of the shortfalls in their subject knowledge. A teacher reports that when *I did the audit on NCETM* (at the start of the programme) *I didn't realise at the time I didn't really understand what the questions were asking me* (Int3 21:29.7 - 2:19.9). She says that she has since gone back to the audits and recognised that she now has a much deeper understanding and insight after two years engagement in the programme. Another teacher makes reference to the concept of proportionality in a similar vein, *Proportion and ratio for me is really eye opening because I hadn't*

realised how little I understood it (Int1 19:47.1 - 19:58.7). It is not perhaps possible at the commencement of a two year programme to have an appreciation of the subject knowledge that might be needed and might be developed.

Summary of Findings in relation to Question 1

The sections outlined above indicate that changes did take place in relation to teacher subject knowledge and these changes formed a particular type of subject knowledge. This is triangulated through its strength both in the questionnaire and interview data. Teachers' subject knowledge is portrayed as deep subject knowledge which is connectionist and relational in form *I am much more aware of making children see the connections because I am much more secure in my subject knowledge* (Int1 20:23.1 - 20:30.1) *What has changed in my subject knowledge is knowing why that worked* (Int1 19:14.0 – 20:01.0), a reference to relational knowledge. There is a noticeable shift in teachers' views of what subject knowledge is. *I thought subject knowledge....was to be able to divide, and because I could divide I could teach division, and the same for the other operations, but that was completely wrong because that was just scratching the surface, it's about having that knowledge of why you use a method and how you use the methods, and all the different methods you can use, not just that you can do it and that's fine and that's how I felt I was as a teacher and how I was conveying it to the children.....so I feel my subject knowledge has got deeper* (Int6 16:10.6 – 16:46.0). The data indicates that for many teachers, this form of deep subject knowledge did not exist prior to the programme and indeed they had not realised it was there to be developed. The nature of the subject knowledge developed was deep, connective and relational in nature. Teachers view this form of knowledge as necessary for effective teaching of mathematics. This will be discussed further in section three of this chapter when consideration is given to the interrelationship between subject knowledge and practice.

Question 2: How does teacher knowledge develop and what factors influence its development?

Question 7 of the questionnaire was specifically designed to answer this question and seems a good starting point to develop insight; however indications of how development took place are also found throughout the data. The data suggests that teachers valued the opportunity to meet with each other and this idea is strong within responses to question 7. Regular meeting enabled teachers support each other. *The network meetings have allowed me to talk to other people in the same situation and share ideas. (Q1 Q7 L3) Opportunities for professional dialogue with colleagues...has allowed us to reflect on issues together and find joint ways of overcoming barriers/challenges within our schools. (Q1 Q7 L10).* Interview 1 has also strong indications that networking and having contact with other teachers who shared the same ethos was very much valued by teachers and one year on from the programme the teachers reflect how they *do miss not attending some of the sessions and having discussions with likeminded people who are really interested, although our staff are interested there is nobody who has the same enthusiasm as I have for the subject. (Int1 11:52.6 - 12:47.8).* Another says: *It would be nice to have another MaST teacher who you could talk to and continue that process of learning together. (Int1 12:47.8 - 13:33.9).* This process of *learning together* seems significant to their development.

Teachers also valued **academic study** and this seems to have contributed to their development, in particular access to research literature. This not only provided them with information, but also developed confidence in their role as subject leaders and champions of mathematics. *The programme as a whole has improved my confidence personally as a maths champion and as a class teacher. Theoretical knowledge gained has been key to underpinning this improved confidence. (Q1 Q7 L12).* This is triangulated within the interviews *I think it's given me the confidence in working with my colleagues because research backs it up.....and the fact that I can say that this is a good method because...I've got the support behind me (Int3 3:57.8 - 4:26.7).*

The **five big ideas** are also seen as influential to teacher development. Responses to question 7 references to the big ideas 32 times as factors which influenced teacher development; *the 5 Big ideas impacted most and altered my thinking/teaching* (Q2 Q7 L12). It is reported that *the content was so easily seen through the 5 big ideas and could be developed through pedagogy* (Q1 Q7 L15). The five ideas appear to have been a connecting vehicle through which development could take place; *the way the course was structured helped me to think about maths in the terms of the 5 big ideas and how they are related. By taking each in turn, I felt that I was able to strengthen my understanding of each aspect and make connections* (Q2 Q7 L22). These findings are also found in the interview data thus providing triangulation, for example a teacher talks about how the five big ideas provided her with a *focus* through which mathematics could be explored (Int7 22:33.1 - 22:57.1). It would seem that seeing mathematics through the structure of the five big ideas was a new experience for the teachers *I don't think I had that idea of the bigger picture of maths before this course* (Int2 1:15.7 - 1:29.7).

The **pedagogic constructs** as outlined in Chapter 1 are also cited by teachers the question 7 as being influential on their development. A teacher reports that *the emphasis on the big ideas and course pedagogies, threading through all the areas of maths, has had the biggest impact in that the emphasis is now on how learning takes place as much as in what learning's taking place* (Q1 Q7 L21). It would seem that the two aspects central to teachers role of mathematics are the *what* and the *how* and the course pedagogic constructs provide the *how* element of teaching and learning. The importance of a focus on specific pedagogic constructs is also present in the interview data *because we homed in on particular thingsfor example what's the same what's different and questioning, prompting thinking through questioning it becomes more of a habit so their (pupils) knowledge is deepened and so is yours by using those pedagogies because they were present in the programme, you did give them quite a lot of focus which meant you understood them more* (Int1 17:21.2 - 18:06.8).

The importance of a focus on the development of **subject knowledge** and actual engagement in mathematics is cited in question 7 as being influential on teacher development *the fact that the programme included the development of subject knowledge alongside the pedagogy of teaching and learning maths* (Q1 Q7 L40) and *the chance to tackle mathematical problems for oneself* (Q1 Q7 L46) This idea is also strong within the interview data *If we had been taught like this I would have understood it years ago.... made those links before* (Int1 5:44.5 - 6:28.4). *We don't feel we've been taught something, we feel we've been enabled to learn it* (Int1 7:16.2 - 8:06.1).

Summary of the findings in relation to Question 2

Teachers' subject knowledge did develop and was informed by several influencing factors as outlined in responses to question 7 of the questionnaire and backed up and developed through the interview data. A masters' level programme engages teachers in academic study and exploration of research literature, it would seem that this has provided insight and developed confidence that they apply to their professional practice, particularly in their role of working with other teachers in their schools. The five big ideas provided an overarching structure in which development could take place and was a new and connective way of looking at mathematics. Engagement in mathematics and the development of subject knowledge was important to the teachers and the pedagogic constructs as outlined in chapter one provided strategies for *how* the learning of mathematics might take place.

Question 3: In what ways does the development of subject knowledge interact with and influence the practice of teachers?

There is strong integration between subject knowledge and teachers professional practice. Teachers do not separate the two and in their reporting through the questionnaires and conversations within the interviews move quite freely between one and the other. There is strong evidence that they have made changes to their practice as a result of the development of their

own subject knowledge. Their own experience in learning mathematics through the programme then influences how they teach it. Key experiences that teachers engaged with which then transfer into practice are explored below for example the role of talk, the use of questioning, looking for pattern, generalising and making connections. Also changes to the nature of their subject knowledge have a profound impact on *how* they teach mathematics in the classroom. In addition to these key ideas that were explored, in particular the *five big ideas*. All of these aspects are explored in more detail below

Talk

The teachers indicate that a significant change to their practice is a greater emphasis on talk in their classrooms to support learning in mathematics (Int1 5:53- 6:30). When probed as to why this has occurred they referred to their own experience on the course and how when their own engagement in mathematical talk supported the development of subject knowledge, as one teacher put it, *it moved my thinking on* (Int1 6:36 - 6:57). There is an indication that engagement in mathematical talk has provided insight into its benefit and motivated them to provide opportunities for the children in their schools to engage in talk. (Int1 3:23 - 4:04)

One teacher talks about the importance of listening to children talk and how she now gives children more time to explore mathematics and listens to their talk. She finds this a valuable form of assessment and says that *you can see how well they understand something* (Int16:57- 7:41). Another teacher talks about how children experience *getting deeper and deeper into the maths as they talk, and they want to keep talking about it* (Int1 6:57 - 7:41). The development of mathematical thinking is linked to the talk and teachers indicate that this is what learning mathematics *should be about* (Int1 8:10 - 8:29). A further teacher talks about using *deeper questioning - drawing out from the children their mathematical thinking and reasoning, rather than assuming that I know what they mean!* (Q1 Q5 L10)

Effective use of talk partners is mentioned and in particular creating the opportunity for children to make conjectures (Int4 18:35.4 - 19:04.3). A teacher talks about conjecturing and convincing being important because *their ideas are not lose, they have to form their ideas tightly and they have to have a reason for that idea* (Int4 20:07.1 - 20:34.8). This she claims is having a significant impact on children's mathematics because they have the opportunity to *challenge each other and build on each other's ideas, which is really strong for them* (Int4 20:07.1 - 20:34.8)

Giving time for talk is important; *using the course pedagogies has 'allowed me' to give children time to explain their own thinking; I had always previously felt under pressure for children to produce pages of written solutions in a silent classroom* (Q1 Q4 L25).

Questioning

The programme identified specific questions to support teachers in developing pupils' mathematical thinking through talk. They were also used with the teachers to develop their own mathematical thinking. The questions of "What do you notice" and "what's the same, what's different" have been influential on teachers practice these *are opening their (the pupils) eyes to so much more, things that were in front of them before are now being seen with clarity* (Int4 20:07.1 - 20:34.8). This is an interesting thought as it seems to reflect teachers own development. They developed a deeper understanding and it became natural to use the strategies in their own teaching to stimulate and deepen children's mathematical thinking (Int1 17:21 - 18:06).

The nature of subject knowledge developed influences the nature of practice

The focus on the development of subject knowledge which embeds mathematical thinking has altered approaches to teaching, *trying to incorporate more mathematical thinking has altered the structure of the lessons, often the pace has been reduced to facilitate the thinking, and there is*

much more emphasis on questioning (Q1 Q4 L23). Another teacher reflects that focusing on mathematical thinking when you are teaching changes the way you approach planning the lessons and delivering the lessons and the children just learn so much more and seem more engaged (Int3 1:39.8 - 2:09.0). Its role in developing greater depth is recognised, mathematical thinking, it's not just about looking at the basics.....but much more depth (Int3 3:21.1 - 3:28.4) Mathematical thinking is influencing the development of pupils. I am now creating independent learners and thinkers so that when they reach a problem that they are unsure of or get 'stuck' that they can think about what they need to do to work through the problem- relational thinking. Of course we know that scaffolding is an important pedagogy but there is a fine line between scaffolding children's learning and spoon feeding and doing their thinking for them (Q1 Q4 L19). Mathematical thinking is at the heart of maths teaching and learning (Q1 Q4 L35)

The 5 big ideas form a structure for the development of subject knowledge and pedagogy

The framework of the five big ideas has been influential both within the development of teacher's subject knowledge and also their pedagogy and the two interrelate. I think it's been a really good way to structure it because....those things being so interlinked and whenever I'm teaching....it's almost like I've got invisible think bubbles in my head that remind me of the big ideas and picking it up in my lessons and when I'm planning, it makes planning so much easier (Int1 30:20.9 - 31:26.2)

Connective knowledge impacts on practice

The connection is made between teachers own personal development and their practice I more overtly make the links for children and find that children will make links back (Int1 19:55 - 20:14).

The same teacher reflects on how this ability is due to the development of her subject knowledge; *I am much more aware of helping children see the connections because I am more secure in my subject knowledge (Int1 19:55 20:14).* A teacher talks about having the confidence to explore negative numbers with Year 2 children. She makes reference to being challenged through the

mathematics that she engaged in whilst on the programme and made to think more deeply. This has given her the confidence to let go and let children explore and handle whatever happens (Int1 20:59.5 - 21:55.5).

Subject knowledge and pedagogy are brought together when a teacher says *understanding the pedagogies behind the teaching of mathematics helps you to develop your own subject knowledge in a way, you are exploring with the children* (Int1 22:07.8 - 22:36). This approach to teaching is contrasted with an instrumental learning approach which the teachers believed they experienced in their own schooling and this, a teacher claims, results in a lack of confidence (Int6 15:23.6 - 16:10.6). Another argues that many teachers feel they should teach in the way they were taught. She argues that where children are taught in a relational way, children are able to make generalisations and this leads to more sustained learning. She says children....they hang on to it if they have the relational understanding, they make the generalisations.....without that depth of pedagogic knowledge the subject knowledge is not going to stick (Int1 22:36 - 23:16).

Knowledge of pattern and generality impacts on practice

The concept of generality, was an area that teachers *hadn't given much thought about before the course* (Int4 17:44 - 18:18), but developed as part of their subject knowledge. The quote from Mason *a lesson without the opportunity to generalise is not a mathematics lesson* (Mason et al., 2005 p1X) has remained with many of the teachers interviewed (Int4 17:44 - 18:18) and is referenced to.

Reference is made to the *always, sometimes, never questions* (Int3 3:28 - 3:36) and how these are regularly integrated into teaching to support generality, providing the opportunity for pupils to make the links and see the relationship between things. A teacher makes reference to Key Stage 2 tests and indicates how developing the ability to generalise can help children answer unfamiliar

test questions. She identifies generality as *the essence of problem solving* (Int1 25:36 - 25:55) and goes on to say that previously problem solving had been *compartmentalised rather than free ranging*. Another makes reference to how the term generality and surrounding language gives children *the language to talk about their maths*. The concept of pattern and looking for pattern is referenced to as an idea that *underlies a lot of maths* and children *became hooked on rather like a drug, they want to find the pattern* (Int1 26:22 - 26:54).

Knowledge of representations impact on practice

Representation features in both the development of teacher and pupil subject knowledge. For example a teacher talks about how it has had a far greater impact than she thought it would (Int4 19:04.3 - 19:37.4) representations have been used to *help children understand the maths*. Many schools seem to have bought additional resources as a result of the programme. Which has also supported them in making informed decisions about what to buy *because sometimes you can buy things that are not actually that useful* (Int4 19:37.3 - 20:07.2)

Summary of the findings in relation to Question 3

As mentioned in the introduction to this section there are clear links between the development of teacher subject knowledge and changes to practice. Changes have been made to their practice as a result of the development of their subject knowledge. Particular influences arise from the nature of that subject knowledge, such as an emphasis on seeking out pattern, generality and making connections, supported through the use of mathematical representations. Pedagogic constructs that teachers experienced in learning mathematics such as talk and questioning also characterise changes to practice.

Sustainability of Change

This chapter has considered and evidenced change, changes to teacher subject knowledge and changes to practice. An important issue still to consider is sustainability of change. For this I will

focus specifically on interview 1, which was conducted one year after the completion of the programme. The changes in the practice and beliefs that the teachers from the interview reported were common with changes reported from across the data and discussed above. I asked these teachers whether they thought that the change is sustainable. They indicated that they believe it is sustainable for those who have done the course. *I would never go back to teaching maths the way I taught it before.* However they did indicate that what is *really hard is keeping it sustained within your school* (Int1 9:40 - 10:26). One reason given for this was changes to staffing and also less time provided for inset in contrast to the time they had been given whilst on the MaST programme. One teacher indicated that she would love more time for staff to do mathematics together. However they are attempting to continue the development and one teacher has recently begun lesson study (Lewis et al., 2006) in mathematics across her school.

I asked whether they believed the change in them is sustainable and there was a definitive 'yes' from all involved (Int1 11:46 - 11:50). One teacher says *It would be very hard to change back again....I don't think it would be possible* (Int1 11:50 - 11:58); another says *you would be nutty to change back* (Int1 13:51.9 - 14:13.1). There are various reasons given for this, one is the improvement of outcomes in children's learning and increases in attainment. Another is children's increased enjoyment in learning mathematics. They identified that the changes are in line with their deeper knowledge of how children learn mathematics and the fact that what they are doing is working *and why would you change something that works?* (Int1 14:13.1 - 14:40.9). They also make reference to other contributing factors such as writing assignments and that the research and reading involved has provided the opportunity to reflect and learn. The long duration of the programme has enabled them to embed practice and witness progress in the children's learning. The time element of doing the programme over two years appears to be a feature in terms of sustainability; *because we were doing it over two years, you actually saw the progress in the children you were teaching* (Int1 15:01- 15:55)

Summary of Chapter 4

This chapter has presented the findings from the data in relation to the research questions. Key mathematical topics are identified in terms of teacher development, but more importantly the nature of the subject knowledge developed, this is described as connective, relational and deep. Factors that influenced this development include seeing the bigger picture, a focus on mathematical thinking, the use of representations and “doing” mathematics, engaging in talk and interacting with likeminded professionals. There is a clear impact of developed subject knowledge on practice. All of these will be analysed in more detail in the next chapter, chapter 5.

Chapter 5: Discussion, Analysis and Development of Theory

In this chapter I will discuss and analyse the findings from the data in an attempt to gain insight and shed new light on answers to the research questions, which are:

- What is the nature of the subject knowledge developed by teachers engaged in the professional development programme?
- How does teacher knowledge develop and what factors have influenced its development?
- In what ways does the development of subject knowledge interact with and influence the practice of teachers?

Research Question 1: What is the nature of subject knowledge developed by primary teachers engaged in the professional development programme?

An exploration of the development of teacher knowledge has revealed several important aspects: teacher knowledge is not necessarily higher, but rather deeper; listening knowledge was a new form of subject knowledge; representational knowledge is important as is the ability to generalise mathematical concepts.

Deeper not Higher

Defining what it is that primary teachers need to know in order to effectively teach mathematics is complex, as discussed in Chapter 2. A key finding within my data in terms of the participants' development of subject knowledge was not that they necessarily had more knowledge in mathematics at the end of the programme. They had not for example learnt any new mathematical concepts, such as trigonometry or calculus; but instead the knowledge they had developed was deeper in nature. The term depth occurs many times in the data, as discussed in Chapter 4 and is characterised as connective and relational; understanding not just the *that* and *how* but also understanding the *why* and how one piece of mathematics connects to another. It was a move towards that type of knowledge that Chinese teachers had in Ma's study (Ma, 1999). Ma characterised this type of knowledge as profound fundamental knowledge of mathematics.

The three forms of mathematical knowledge in terms of *that*, *how* and *why* are described by others as the factual, procedural and conceptual (Voutsina, 2012., Schneider et al., 2011., Rittle-Johnson et al., 2001) and all three are seen as essential for successful learning in mathematics. For example I know *that* $7 \times 8 = 56$, I know *how* to find the area of a rectangle and I know *why* the formula for finding the area of a rectangle works, which enables me to apply it to more complex shapes. It is in particular the last of these three forms of knowledge that has been significant in the development of the teachers within my study as evidenced within the data and forms the basis of deep conceptual knowledge. The first two, many teachers already had or could quickly acquire if needed, it is the third that is the more challenging and the teachers in this study were shown to develop. Several made reference to the fact that it was this type of knowledge that was missing from their own education. The findings in Chapter 4 suggest that the third form of mathematical knowledge, the *why*, can be characterised as *deep* subject knowledge and is essential to teaching. The parameters set nationally for the MaST programme included the development of deep subject knowledge. There was however no interpretation given as to what this meant, the teachers themselves seem to have defined this in terms of their own development. The teachers define this depth in terms of relational knowledge as defined by Skemp (1976). They also discuss it within the context of making connections (Askew et. al., 1997). Making connections is something they actively engage in when planning and they see relational understanding, the understanding *why* as important in this process.

Often when reference is made to deficiencies in primary teachers' subject knowledge, the inference is that the teachers need to know more mathematics, or they need to get a higher grade at GCSE. The data suggests that the teachers in the study did not necessarily come to know more mathematics but needed to understand more deeply the mathematics that they did know. For example a teacher discusses how her view of subject knowledge has changed, originally she thought it was just about being able to do the mathematics, such as carrying out operation of

division, but she now realises that level of knowledge was just *scratching the surface* (Int6 16:10.6 - 16:46.0). The teachers in the study all had a GCSE in mathematics, but it would appear that their formal qualifications had not developed the deeper form of knowledge describe by teachers in the data. This has profound implications for teacher training and professional development and the question needs to be asked *are all teachers developing this deeper form of subject knowledge?*

Defining the nature of deep subject knowledge is important if it is to be developed. There are some clues within the data as to the form it took. A teacher says *thinking about instrumental teaching and understanding how little it contributes to a deep understanding of maths..... generality, particularly algebra is a much larger area of maths than I had considered. Thinking about algebra as a structure and a framework has changed my own thinking about this area of maths..... Looking at the structure behind many areas of maths has deepened my own understanding* (Q1 Q4 L66). Here deep knowledge is contrasted with instrumental knowledge, suggesting depth is related to relational knowledge, an understanding of the *why* (Skemp, 1976). It also encompasses generality and focusing on mathematical structures. Another teacher says; *it's the connections that's deepened my knowledge, it's ... the fractions and decimals and multiplication and how they all link together under proportionality so I think it's all the linking together of all my bits of knowledge that's developed in depth* (Int6 14:54.1 - 15:23.7). This reflects the *knowledge in pieces* referenced to in the research literature (Orrill and Brown, 2012). This research highlighted that teachers can fail to link existing knowledge together and therefore do not apply the most appropriate knowledge within their practice. The teacher makes reference to proportionality and how this has helped her to make connections between proportional relationships.

The development of deep knowledge has been supported through the use of representations that enabled mathematics to be seen in a way *not seen before light bulb moments* (Int7 32:08.2 - 32:31.0). Depth is characterised by clarity and appears to be accompanied by an excitement at discovering something new, not new in terms of the mathematics, but a new way of seeing it.

Listening Knowledge

A teacher says *I listen more to what the children say because I am trying to find them conjecturing, I'm listening for those conjectures, so that I can say...can you prove it? So I don't think I even heard as much of what the children said before because I wasn't listening, I didn't know it was there to be listened for* (Int7 1:56.9 – 2:21.8). This quotation has provided me with new insight into the development and nature of teacher subject knowledge and brings together a number of threads within the data, such as the importance the teachers give to talk and communicating mathematics and the role of mathematical thinking in learning mathematics, stimulated by thought and interaction. I believe it is significant that the teacher claims that she *didn't know it was there to be listened for*, suggesting that this was new subject knowledge, something she had previously not known. The nature of this new knowledge is to recognise in pupils their engagement in mathematical thought so that the teacher might play a role in promoting and extending it, by saying *prove it*. This developed knowledge forms part of the teachers' subject knowledge, where teachers recognise the importance of particular processes that are both important to learning mathematics, but also important to being a mathematician and operating mathematically. Previously the participating teachers had been oblivious to these ideas *I didn't know it was there*. Within Shulman's (1986) categorisation, this might be categorised as pedagogic content knowledge, however I would argue that it is more than that, it is syntactic knowledge, an aspect of the mathematics. It is not only relevant to the process of learning mathematics but is part of operating as a mathematician and is therefore at the core of mathematics subject knowledge. Mathematics subject knowledge is not just a static body of knowledge, but a way of thinking that

allows the teacher to operate in mathematical ways and foster the development of their pupils to operate in mathematical ways.

The development of listening knowledge was a new idea, something I had not thought much about before. There is reference in the research literature to the need for teachers to really listen to pupils (Davis, 1997, Sherin, 2002) in order to access and analyse their thinking and any misconceptions in order to respond appropriately in and move learning forward. Pepin (2011) identifies possible purposes for teacher listening within the mathematics classroom; to identify understanding and correct pupil mistakes; to access pupil thinking; to actively engage pupils in thinking and negotiation of meaning. Teachers however can only do this, particularly the last point regarding development of thinking if teachers know what they are listening for. Effective listening to support learning in mathematics does not just rely on pedagogic knowledge, knowing *why* it is important, but in having subject knowledge and knowing what to listen. It requires engagement in mathematical thinking as the listening takes place, as reflected in the quotation above.

There is evidence from across the data of an increased use of talk, not just as a means to communicate mathematics, but as a means to learn mathematics. It is often closely allied within the data to the development of mathematical thinking within the process of learning *lots of paired talk to discuss and share ideas to encourage more thinking and reasoning* (Q1 Q5 L82). The importance of talking to mathematical thinking was emphasised by Sfard (2008) who considers thought as a form of communication. The data supports the idea that verbalisation is an external expression of mathematical thought. The construct of mathematical powers (Mason et al., 1982), in particular conjecturing and convincing seems to have acted as a vehicle for talk and as evidenced in Chapter 4 is strong within the data. The data evidences that teachers are listening to pupils in a different way as a result of their developed subject knowledge; *children's ideas must*

be listened to and channelled to lead to greater exploration and understanding of the maths being investigated (Q1 Q5 L43).

The data extends the purpose and benefits of listening to pupils beyond that found in the research literature to suggesting that one of the benefits of listening to pupils reasoning about mathematics is for the teacher to develop and extend their own subject knowledge. Through the process of listening to someone else's thought processes, the listener is engaged in the process of mathematical thinking which provides opportunities for new learning. A teacher reports how *at least once a week the children teach me something now which would never have happened before* (Int7 2:21.7 - 3:17.3). The listening is not a passive activity but an active one which seeks to connect with pupils' mathematical reasoning. Teachers' engagement in listening is to understand pupils' thinking and in doing so, they are cognitively engaged in thinking mathematically. The change in the nature of teachers' subject knowledge has facilitated a dynamic form of knowledge that is actively seeking to make connections in mathematics as an ongoing process. This nature of subject knowledge transfers into practice where teaching and listening to pupils provides an opportunity for teachers to extend their own subject knowledge. When listening to pupils they are listening in a different way, they are listening for things that previously they had not realised *were there to be listened for.*

Representational Knowledge

The importance of representations in learning mathematics is well documented (Ryken., 2009, Turner, 2008, Lesh et al., 1987,). Lesh et al (2007) suggests that five types of representation are available to mathematics teachers: world contexts, manipulable models, pictures and diagrams, spoken language and written symbols. Within the context of my data it is the middle group that the data makes reference to the most with reference to representations in the form of manipulable models, pictures and diagrams (see Appendix 4).

In Ma's research (1999) an important characteristic of Chinese teachers' subject knowledge was an ability to identify multiple and appropriate representations to support the development of pupils' conceptual understanding. As seen within the data the use and impact of representations on teachers' knowledge and practice is strong. However the use of representations to support the teaching of mathematics was not a new idea to them, unlike the concept of generality for example. What changed in relation to the development of their subject knowledge was that they gained insight into mathematical structure embedded within representations and how this might be used to reason about the mathematics and develop insight and understanding (Int3 14:45.6 - 15:16.5). The data indicates an increased use of representations in teachers' practice and also a shift in the purpose of their use with a greater emphasis on using them to help pupils make sense of the mathematics (Int6 24:10.2 - 25:17.4), show mathematical thinking (Int6 23:34.9 - 24:10.2) and develop understanding (Int6 25:59.4 - 26:21.4). Now it seems they are more likely to be used to help pupils reason about and understand concepts. A particular representation is mentioned, that of the bar model (Beckmann 2004) where a teachers says *the Singapore bar, its knowing to break it down and take it back to the basics and building on that and I think that's made me think about all of my practice, how to break it down and then build it up again so they understand* (Int7 9:44.8 - 10:17.5). Representations are being used to break the mathematics down, the word *simplify* is used later in the same conversation.

Representations of mathematics were used extensively on the programme, including physical objects and I filled the boot of my car with them as I travelled across the country to work with teachers on the programme. The data suggests they served a dual purpose for teachers, both as a vehicle for subject knowledge and as a pedagogic tool. A teacher says *representations and they've made it really clear and the amount of times I've sat here and gone Oh yeah that's a really good way and you take it back to the classroom and the children can see it and that also builds your*

confidence as a teacherprepared to take more risks with different kinds of representations and the children will also show you ways that something is clear to them in a different representation (Int6 5:21.7 - 5:46.4).

Teachers make reference to the use of representations in providing insight into the mathematics to develop their own conceptual understanding *I am now much more interested in the mathematical thinking behind the rules and have used representation on numerous occasions to develop my own knowledge and understanding (Q1 Q5 L88)* They also make extensive reference to their increased use in their practice. They serve as a tool to develop their knowledge and also a tool to be used in the classroom to develop pupils' knowledge. *Use of models and images; I now use a variety of models and images to support children's mathematical thinking and am actively encouraging my colleagues to do the same.....the power of reasoning through talk and visual representations. (Q1 Q5 L85).* It seems that providing the opportunity for teachers to experience development of their own subject knowledge through the use of representations was important in supporting their use in the classroom; *So I think playing with the maths yourself....., using mathematical resources yourself and talking to other people about it and seeing the model in yourself and going through it, discussing the representations, discussing the mathematical thinking behind things, that was very helpful.... and I want to keep doing it because that has really affected the way that I teach maths and the way that I encourage the staff at my school to teach maths (Int1 3:23.0 - 4:04.5).*

Teachers recognise representational knowledge as an important component of their subject knowledge and make reference to the importance of representations many times, particularly as an aid to develop conceptual understanding of both pupils and themselves, as outlined in Chapter 4. Knowledge of mathematical representations in terms of their structure and how they represent

mathematical concepts is a key element of teacher subject knowledge. In question 6 of the questionnaire where teachers are asked to identify key areas of mathematics development; representation is the second most recorded item. This key area of subject knowledge has merged with their practice and they view it as mathematical knowledge. The data indicates that they use representations for the purpose of their mathematical structure and their role in developing conceptual knowledge. This is in contrast with the beginner teachers observed by Turner (2008) who selected representations for superficial reasons, such as their attractiveness.

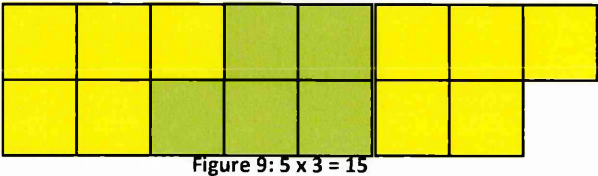
There is some disagreement in the research literature as to whether or not physical representations do support learning in mathematics as it appears to suggest within my study (Pouw et al., 2014, McNeil and Uttal, 2009, McNeil and Jarvin, 2007, Pape and Tchoshanov, 2001, Bills and Gray, 1999,). My data shows that whether or not they are supportive to learning may depend not just on whether they are used, but on how they are used. To be used effectively requires a particular type of subject knowledge on the part of the teacher, as expressed by a teacher in my study; *it's the representation, you need to know, you're bringing your knowledge to the representation, the representation is not going to teach the children itself so you've got to have that subject knowledge to deliver that* (Int2 10:49.9 - 11:04.4). Another teacher provides a specific example involving multiplication; *I know my times tables so I thought I had good subject knowledge and that's what I thought the children had to learn but now I realise I can deepen their subject knowledge by scaffolding because I understand it, like arrays and things, I'd never used those ideas before, because my subject knowledge has changed, I can now scaffold and ask better questions* (Int6 18:33.9 - 19:06.5). It is the scaffolding that is important to enable pupils to gain maximum benefit from the use of representations. This has implications for both teachers own subject knowledge and their pedagogic skill in applying it to support pupils' knowledge. Representations of mathematics, by themselves are not necessarily going to support children's

mathematical development, but it is the subject knowledge and skilful application of the teacher who can draw attention to the structure of the mathematics embedded in a representation.

A change to teachers' practice has been that representations are no longer reserved for younger children, there are 12 specific references in the data to their use with older children *getting the equipment out and doing it in different ways with different representations, even when they are in year six, getting beyond that hurdle that we only use equipment lower down the school... getting everyone using it to link the apparatus with what the concept is and why you're doing the processes and algorithms* (Int6 9:52.2 - 20:25.1). It is seen as important that all children gain a deep and conceptual understanding of the mathematics and representations are seen as a means of achieving this aim. There is discussion in the research literature as to whether concrete manipulatives are only appropriate for use with younger pupils due to their stage of development and the need to operate in the concrete (McNeil and Uttal, 2009). It would seem that teachers in the study saw the value of using them with older children, as a means to extend their mathematics and enable them to achieve more. A teacher reports that *I went into Y6 (bringing resources). At the end I asked them did you enjoy it, did you find it easier or more difficult and.... their reaction was.... they thought it was hard maths but they loved doing it whereas previously hard maths often made children want to give up.....because they could do it (a physical activity), it felt doable* (Int7 19:02.5 - 19:54.8). It seems that the manipulatives used provided access to the mathematics.

There is discussion in Chapter 2 from the research literature of the importance of representational knowledge for teachers. My research has provided detail in terms of the form this knowledge takes and how it is acquired. My research has found that the form this knowledge takes is developing the ability of teachers to see mathematical structure in representations of

mathematics which provide insight into mathematical concepts. For example teachers were given a statement: *the product of two odd numbers is odd. Is this always, sometimes or never true?* It was the representation of the mathematics that enabled teachers to come to a confident explanation and generalisation as a result of seeing the structure of the mathematics embedded in the representation used. An odd number, such as 5 as in each piece of the apparatus represented in Figure 1, replicated an odd number of times will result in the product being an odd number. From the structure of the specific example below comes a generalisation that the product of 3 odd numbers will always be odd due to the mathematical structures involved. Each pair of odd numbers will combine to form an even number and the addition of an extra piece on the end will result in the total (product) being an odd number.



The data indicates that teachers gained significant confidence in being able to see mathematical structures and relationships exposed through the concrete materials and images and recognised the role of representations in deepening their subject knowledge. This knowledge was acquired through teachers engaging with mathematical representations in the form of concrete and visual resources within the process of their own learning. The representations exposed the underlying structure of the mathematics and developed insight and relational understanding. Teachers also reported that they developed knowledge and insight on the course into how to use the resources/representations of mathematics effectively to provide access to mathematical structure for their pupils. Hence it seems that if teachers explore representations in an environment where they are learning then they are sensitised to the power of representations to explicate and deepen mathematical knowledge and thus seek out more and alternate representations to use in their teaching of mathematics.

Typically it would seem that in many schools outside of the Northampton MaST programme representations are not used in the way described within the data as resources to expose mathematical structures and aid conceptual understanding. The data, as discussed in Chapter 4 indicates that this was a new way of working. Lack of attention to mathematical structure is also reflected in the research by Rowland et al (2009) where the representation of a hundred square is used to add 11, 19 or 21, the emphasis by the teacher was placed on the procedure of moving down and left or right on the square rather than to develop understanding of the structure of the mathematics presented within the representation and why the procedure of adding 11 by moving one square down and one square to the right, works. Rowland argues that an alternative representation might be preferable, however I would argue that with appropriate scaffolding and attention being drawn to the structure of the mathematics as represented in the hundred square pupils' reasoning can be stimulated and the conceptual understanding developed. As Delaney says *there is no mathematics in a resource, the learner brings the mathematics to it* (Gates, 2001).

Effective use of representations to support learning in mathematics has implications for teacher development. Firstly teachers need to see the mathematics in the representation themselves and secondly they need to be able to skilfully scaffold pupils so that they can see the structure of the mathematics within the representation. Although the mathematical structure is there, pupils will not automatically see it. This is attested in the data *opening their (the pupils) eyes to so much more, things that were in front of them before are now being seen with clarity* (14 20:07.1 - 20:34.8). The question *what's the same, what's different* is a vehicle that is supporting this process, where two representations are compared. Skilful scaffolding can draw learners' attention to the structure of the mathematics as indeed it did for the teachers on the programme *oh yeah; I've never seen it that way before* (17 32:08.2 – 32:31.0).

Representational knowledge would appear to be an important characteristic of teacher knowledge relevant to their role in developing pupils' subject knowledge. Within the study it has acted as a vehicle for the development of teachers own subject knowledge and within the classroom it acts a vehicle for pupils' subject knowledge, mediated by teachers' subject knowledge in scaffolding its use.

Generality

The ability to generalise is identified within the literature review (She, 2011; Williams, 2008; Ma, 1999; Askew, 1997 Shulman, 1986) as an important feature of teacher subject knowledge. The data indicates that teachers within the study also recognise its importance as indicated in Chapter 4.

The decision whether or not to include generality as a big idea on its own or to combine it with pattern was carefully considered when developing the professional development programme. The decision was made to include the two as separate ideas. The importance of both was recognised, however my experience led me to believe that teachers would be very familiar with pattern as an idea in mathematics but much less familiar with generality and it might therefore be helpful to provide its own distinct focus within the programme in order to develop a deep understanding. Generality was viewed as a step beyond pattern where having recognised a pattern the learner seeks to understand the reason for the pattern by looking at the underlying structure and generalizing the mathematics beyond the examples in front of them.

The quote from Mason which was used within the introductory session to generality *a lesson without the opportunity to generalise is not a mathematics lesson* (Mason et al., 2005) has remained with many of the teachers interviewed (Int4 17:44 - 18:18). It was quite startling to some teachers, that an idea they had not even thought about before could be so important to

learning mathematics. Although they recognised its importance they did find it challenging to develop their own confidence and skills. *It's the one (of the 5 big ideas) that took me longest to get a hold of and still sometimes I think now, have we generalised, have we given them (the children) an opportunity as Mr Mason would say? I do have to work hard and to talk about it with the other teachers, really concentrate and pick out where they have generalized, but it is huge, I can recognise it as very important (Int7 13:26.2 - 14:04.3).*

Teachers connect generality with other ideas, in particular pattern, relational understanding and making connections. The concept of generality has become part of how the teachers in the study think about mathematics and at the same time a core element of their subject knowledge, their propensity to make connections, leads to them recognising pattern and structural relationships which lead to generalisations.

Research Question 2: How does teacher subject knowledge develop and what factors influence its development?

It is not sufficient to know what subject knowledge primary teachers need in order to effectively teach mathematics; if it is to be developed then it stands to reason that it is necessary to understand how it might be developed and effective strategies for doing so. Some of the elements that have emerged from the data were expected and some were a surprise, things that I had previously not thought of, thus developing my own insight and understanding and adding to the research literature. Key ideas that have emerged from my analysis of the data are that subject knowledge is not just a body of knowledge but a way of thinking; the five big ideas which formed the structure of the professional development programme facilitated the process of making connections; engagement in and reflection on mathematics developed the way teachers thought about mathematics and this was also supported by academic reading; opportunities to network and develop alongside other teachers were also influencing factors in teacher development.

Subject knowledge develops as a way of thinking

Subject knowledge is traditionally identified as content knowledge (Schulman 1986) and thought of as a list of content to be learnt, understood and applied. However the data suggests that for the teachers involved in the study it is more than this. It is in fact a way of thinking about mathematics, which influences how they learn and what they learn. It seems that their thinking about mathematics has changed, influencing profoundly their approach to the development of subject knowledge in the classroom. These changes to thinking about mathematics are not just ephemeral aspects but are rooted in subject knowledge. Teachers' desire to make connections for example is reliant on conceptual understanding, but goes beyond understanding concepts to wanting to make connections between them. The desire for relational knowledge goes beyond knowing the mathematics to understanding why techniques and procedures work. The importance of seeking out generalisations goes beyond understanding individual content to seeking out the underlying structure of the mathematics and seeing it as the same for multiple contexts.

Shulman (1986) touched upon the complex nature of subject knowledge for teaching in his distinction between substantive and syntactic knowledge in relation to content knowledge, which he developed from Schwab (1982). Substantive knowledge is knowledge of *the variety of ways in which the basic concepts and principles of the discipline are organized to incorporate its facts* while syntactic knowledge is knowledge of *the ways in which truth or falsehood, validity or invalidity, are established* (Shulman, 1986) (p9). Within the context of mathematics I have interpreted substantive knowledge as being the body of mathematical knowledge, its concepts, facts, rules and techniques; and syntactic knowledge to be the key processes involved in being a mathematician such as engagement in mathematical thinking in the form described in the literature review, leading to generalisation and proof. Within my study, it is particularly the development of syntactic knowledge which is new to the teachers; however there is a two way

relationship between the two where syntactic knowledge is contributing to the development of substantive knowledge. Teachers' engagement in mathematical reasoning, conjecturing, convincing and seeking out connections is developing their understanding of mathematical concepts. This form of subject knowledge is influencing their development of subject knowledge in a way that has greater depth. They are not just learning mathematics, but developing the characteristics of mathematicians. It would seem that these changes to thinking are the driving force for how their subject knowledge continues to develop and how they apply and continue to develop their subject knowledge within the context of their practice. This is discussed further in Section 3.

The Influence of the big Ideas and making connections

Seeing the bigger picture of mathematics and how it connects together, particularly through the framework of the five big ideas, as discussed in Chapter 1, would seem to have influenced the development of teacher subject knowledge. Reference to them in the data is strong, as described in Chapter 4. Their influence on changes to teachers' thinking and practice is summed up by one teacher who says they are *forming a better structure and, umbrella for teaching.....rather than thinking OK its 4 operations and that's what we're focusing on, you're actually not you are focusing on the mathematical thinking or the generalisation all these other things that we've looked at, so regardless of what area you are covering, its more about those skills, rather this is the one thing I want you to remember* (Int3 2:09.0 - 2:35.3). The big ideas seem to provide a connective framework to bring ideas together. Another teacher makes a comparison between her recent development of subject knowledge within the programme and a previous masters' module she had engaged in and recognises that it had been far greater in the programme under discussion. Reflecting on why this might be the case, she says *I think it's the depth to which we go, looking atthe concepts and the skills..., putting it in one place, through those big ideas* (I1 0:23.5 - 1:15.8). Another recognises that the big ideas approach helped in *seeing all the*

relationships within mathematics (I2 1:15.7 - 1:29.7). It would seem that the framework of the big ideas has influenced the development of teachers' subject knowledge particularly in providing greater depth of understanding and the ability to make connections. A teacher reports that a focus on *5 big ideas helped me to focus in* (I1 27:24. - 27:34.). They bring ideas together, one idea supports another, developing generality through representations for example or developing understanding of proportionality through mathematical thinking and all support not only learning but teaching of mathematics. It enables teachers to see mathematics in a more holistic manner and connect it to their teaching.

Overall the professional development experienced by the teachers engaged in the programme appears to have been of a connective nature. Reference to making connections is strong within the data as discussed in Chapter 4. There are two features to this: making connections within mathematics, structural connections within and between concepts; and also making connections more generally, between the big ideas, and between subject knowledge and practice. A focus on just a small set of key ideas and the revisiting of them over the period of two years in the form of the big ideas, as discussed above appears to have been an influencing factor and developed the depth needed to make connections.

Development is experiential and reflective

Teachers report that the experiential nature of the programme where teachers engaged in mathematics as learners was influential on their development in several ways. A teacher reflects on the fact that they were placed in a similar position as a pupil and able to reflect on the process (Q1 Q7 L86). The experiential nature also enabled teachers to experience pedagogy at the same time as developing their subject knowledge, which is an important learning opportunity for teachers. As in Ball's (Ball, 1990) research it provided an opportunity to break with experience and

experience learning mathematics in a different way to that which they had previously been taught.

Teacher's engagement in doing mathematics as part of the programme facilitated the process of mathematical thinking. Previous research has linked a focus on mathematical thinking as an influencing factor on successful and sustainable professional development (Swan and Swain, 2010, Warren, 2009, Steinberg et al., 2004). It seems to have had a profound influence also on the teachers engaged in my programme, as evidenced in Chapter 4. It now defines the way they think about mathematics and influences their development of subject knowledge, how they learn, what they learn and how they act in the context of practice.

The way teachers think about mathematics and their ability to reflect was also influenced by their academic reading. Teachers' engagement with academic literature, including both theory and research literature is referenced to in the data and equates to De Geest's (2011) research in that it gave teachers authority and confidence. In addition to this it influenced the way in which they engaged in mathematics, and developed their subject knowledge, a teacher makes reference to an influencing factor on her development was *the chance to tackle mathematical problems for self in the light of Mason, Gattegno and Skemp* (Q1 Q7 L47).

On reflection I did make a conscious decision to select research literature with some mathematics in it, for example Miyakawa and Winslaw (2009) where teachers had the opportunity to engage in an activity involving proportional reasoning. This experience I would argue provides a more dynamic experience for engagement with research. I had not thought in detail before the influence that theory might have within the process of doing mathematics, it has given teachers a deeper insight and I believe influenced the distinct way of thinking about mathematics that is strong within the data.

Networking

The literature review makes reference to communities of practice (Wenger, 1999). It would seem from the data presented in Chapter 4 that such communities emerged and were supportive to teacher development within the programme. As evidenced in Chapter 4, there is significant reference to networking together and teachers supporting each other. There was a desire from amongst the teachers that they would value continuing to meet, however the evidence from Interview 1 involving teachers one year after the programme, suggests that this has not happened *I do miss not attending some of the sessions or having that time to have those discussions with likeminded people.....no one who has the same enthusiasm for as perhaps I have for the subject and I just miss having those chats* (Int1 11:52.6 - 12:47.8) The University and Local Authorities occasionally bring their MaST teachers together and these events are well attended. I have set up a community on the NCETM website open for MaST teachers nationally, not just my own. There are several hundred registered but the interaction is minimal. It does not seem to be able to replace the regular face to face meetings with the same group of people that they experienced whilst engaged in the programme. It would appear then that an important aspect to my continuing practice is to consider how to make the facilitation of networking of teachers sustainable over time.

Research question 3: In what ways does the development of subject knowledge interact with and influence practice?

The relationship between subject knowledge and practice is a crucial focus for fully understanding teacher subject knowledge. It is essential that teachers' subject knowledge is viewed within the context of their practice for it is here that its purpose is played out. The data closely aligns the two; teachers almost always talk about their subject knowledge within the context of their practice as evidenced for example within the questionnaire data where the answers to questions

four and five are very similar, even though one is about teachers own learning and the other is about their practice. In this section I will discuss three aspects which have emerged from the data:

- the interrelationship between subject knowledge and practice,
- the development of subject knowledge within the act of teaching and
- pedagogic vehicles for the transfer of subject knowledge into practice

Subject knowledge and pedagogy are bound together

There is evidence of a connection between how teachers learnt mathematics in the development of their own subject knowledge and how they in turn develop children's subject knowledge; *seeing the benefit I've had when doing talk when doing mathematical activities and how useful it's been when we've been working on something and how that's helped me to move my thinking on, and then giving children that opportunity to discuss* (Int1 6:36- 6:57). Another teacher talks about how her subject knowledge became connectionist and she now applies this in her teaching *I am much more aware of helping children make the connections because I am much more secure in my subject knowledge* (Int1 19:55.5 - 20:14.5). As discussed previously Ball (1990) talked about how strong the tendency is for teachers to teach in the way that they have been taught and in order to break the cycle and prevent teachers reverting to the teaching of their own school days it seems to be important for professional development to be experiential and for teachers to experience a different way of learning mathematics in order for it to impact on practice in a sustained manner. The experiential nature of the programme and its subsequent impact on practice is strong within the data, as discussed in Chapter 4.

For teachers it is difficult to separate subject knowledge and practice, the two are inextricably linked within the context of teaching. The data demonstrates that the development of teachers' subject knowledge informs their practice on a daily basis. For example in the context of a teacher's selection of a mathematical representation to support pupils learning of a mathematical

concept, teachers' subject knowledge will influence the selection (Int6 18:33.9 - 19:06.5). There is subject knowledge involved in the identification of the structure of the mathematical concept in the representation to support learning

Also for the teachers within my study their pedagogy was subject knowledge informed, the use of talk for example was used to promote mathematical thinking, as are the questions they ask, such as *What's the same, what's different, or what do you notice?* Their purpose is to make mathematical connections, an important aspect in the development of their own subject knowledge.

Heggarty (2000) argues that the effects of teacher knowledge can only be understood within the context of teaching. Mason and Spence (1999) argue that teacher knowledge is one thing as a static body of knowledge but knowing how to act in unique classroom contexts is another, thus they argue for a dynamic form of knowledge that is able to bring a variety of types of knowledge together in a given moment and act appropriately. Teachers in my study seem to recognise this requirement. *I think teachers themselves need to have the flexibility of thinking as well, they might be used to knowing where the lesson is going to go and actually have got to be able to adapt it...depending on what comes up in the lesson....and what the children come up with, I think that involves a deeper level of understanding that maybe some teachers currently don't have* (Int3 15:54.2 - 16:19.0). It is interesting that the term depth is used to describe this dynamic form of teacher knowledge. Depth is a common theme throughout the data and was defined as a relational, connective form of knowledge evidenced in Chapter 4 and is applied to interact with practice in a dynamic way to support the learning process. Knowledge is built up over time, often in discrete nuggets of information (Orrill and Brown, 2012) the ability to make connections between the pieces and be able to apply these to a diversity of contexts is an important characteristic of teacher subject knowledge.

The argument has sometimes been made that for primary teachers the most important thing is pedagogic skills, that once you have these, you can teach anything within the primary curriculum, the rest is just facts to be acquired. The teachers talk a great deal about their subject knowledge and the nature of this is discussed elsewhere. It is clear that their conceptualisation of subject knowledge has moved beyond knowledge of facts and procedures to a deeper conceptual form of subject knowledge. For their pupils to develop this form of knowledge requires a new form of pedagogy where subject knowledge and pedagogy are inextricably bound together. They come together in knowing how to act in the moment within the context of practice, knowing the questions to ask, the lines to pursue. The programme has given teachers pedagogic tools to support this process and it has developed not only their mathematical knowledge, but ways of thinking in mathematics which are dynamic and are applied within the act of teaching. Subject knowledge for teaching does not sit outside the act of teaching but is inextricably bound to it. Pedagogic and subject knowledge have not only developed side by side but have integrated, developing a new form of pedagogy which is subject knowledge informed. This new form of pedagogy is characterised by a desire for children to reason and think mathematically. Mathematical thinking is extremely strong within the data, as evidenced in Chapter 4. Teachers' pedagogy is driven by the desire to facilitate and promote mathematical thinking within the process of learning mathematics, they are teaching in a different way and this way is informed by their developing subject knowledge.

There is evidence that teachers recognise a relationship between the development of subject knowledge and the pedagogy they apply to practice *understanding the pedagogies behind the teaching of mathematics helps you develop your own subject knowledge, in a way, you are exploring with the children. If you've got good subject knowledge and you don't understand the pedagogic side of things, you're just trying to fill those children up with knowledge, rather than*

letting them explore and letting them investigate (Int1 2:07.8 - 22:36.0). Another teacher reflects that without the *depth of pedagogic knowledge subject knowledge is not going to stick* (Int1 22:36.3 - 23:16.7). Teachers appear to have adopted a particular form of pedagogy that matches the development of their own developed subject knowledge.

The link between subject knowledge and pedagogy has implications for teacher professional development, should subject knowledge and pedagogy be taught separately or together? Practice both in this country and elsewhere is variable (Adler and Davis, 2011). I would argue that because of their interrelationship they should be taught together, there is an inextricable link between them, and maybe not even a link, but maybe both part of the one concept. Teaching mathematics requires the teacher to think and act in mathematical ways, the way they now act as a result of development of a particular form of subject knowledge.

Teachers subject knowledge develops within the act of teaching

A new element, not found in my reading of previous research, has emerged from my data regarding the continued development of teacher subject knowledge within the act of teaching. This is mentioned within the context of *listening knowledge*, explored earlier in this chapter. There is evidence that teacher subject knowledge is continuing to develop within the act of teaching and this appears to have resulted from changes to pedagogy which in turn has resulted from changes to the nature of the subject knowledge developed by teachers. Changes to the nature of teachers' subject knowledge involved it becoming relational, connective and conceptually deeper. They reason about and look for connections in the mathematics and this continues into their practice. They expose opportunities for pupils' subject knowledge to go deeper through reasoning, conjecturing, convincing and exploration as demonstrated in the findings in Chapter 4. This makes greater demands on their subject knowledge and provides opportunity for its development to continue through the dynamic classroom context where

mathematical thinking and the associated processes of reasoning, conjecturing and convincing are applied within the process of learning. This represents a change to classroom practice *The big idea that's been vital to my teaching is mathematical thinking, for our children.....they expect to be spoon fed, and they don't want to reason, they don't want to think but that has completely changed and turned on its head* (Int6 26:44.3 - 27:30.7).

There is significant evidence (80 references) across the data that teachers' pedagogy has shifted to a more investigative exploratory approach, for example *I now plan much more investigative work in my maths lessons which gives the children opportunities to explore different concepts and develop their mathematical thinking and reasoning skills.* (Q2 Q5 L71). This has resulted in a classroom climate which is conducive to both pupils and teachers reasoning, making connections and discovering new depths. For example a teacher talks about children in her own class who represent the mathematics in a way that she has not seen before, which is a *light bulb moment.... I've never seen it that way before* (Int7 32:08.2 – 32:31.0). On reflection I feel that this is also true of the development of my own subject knowledge, it is continually developing through my teaching and interaction with teachers. In doing mathematics with them and scaffolding their learning, I too make new connections and gain fresh insights.

From the data has emerged a cyclical relationship between teacher subject knowledge and practice. I have developed a model to illustrate this relationship in Figure 2. Teachers' subject knowledge has developed as connective and relational in nature and embodies reasoning and mathematical thinking. This in turn creates in teachers a desire for their pupils to develop such knowledge, which in turn requires a need to apply a particular form of pedagogy. This particular form of pedagogy creates a classroom climate which is conducive to engagement in reasoning, mathematical thinking and making new connections on the part of both pupils and teachers. This in turn develops not only pupils' subject knowledge but also teachers' subject knowledge.

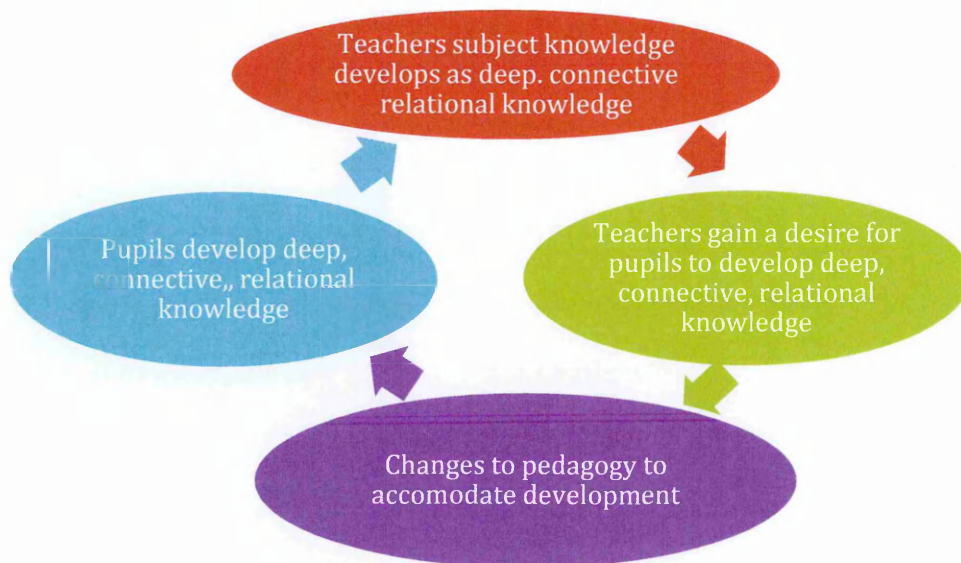


Figure 10 Cyclical and Dynamic Nature of the Development of Teacher Subject Knowledge

Pedagogic vehicles for the transformation of subject knowledge into practice

The purpose of the development of the subject knowledge of the teachers engaged in the Northampton MaST programme was that it should transform practice and make them better teachers of mathematics. There is a great deal of research on the nature of mathematics subject knowledge as discussed in Chapter 2 but much less is discussed in the literature as to how to develop subject knowledge, such that it has a positive influence on practice. The data from my study suggests that there were effective vehicles used within the context of the professional development that did support transformation and application of subject knowledge into practice.

Two key pedagogic questions were embedded into the design of the Northampton MaST programme as discussed in Chapter 1: *what do you notice* and *what's the same, what's different*. The data indicates that these two questions had a significant influence on teachers practice and provided for a connective form of subject knowledge to be applied. The first directed task for participant teachers was to ask these questions in their mathematics lessons and reflect and feedback on the impact. Particularly the question *what's the same, what's different* is very strong

within the data with sixty references, 12 of which are from the interviews and spread across seven of the eight interviews. One teacher describes it as a catalyst for change alongside a focus on relational learning (Skemp, 1976) *when we had to look at the Skemp article and the power of what's the same, what's different.....they really got me thinking about how I'm doing things and I think that was like a catalyst for other things* (Int1 1:23.6 - 1:50.0). Another says *because we homed in on particular thingsfor example what's the same what's different and prompting their (pupils) thinking through questioning, it becomes more of a habit so their knowledge is deepened and so is yours by using those pedagogies* (Int1 17:21.2 - 18:06.8). The first reference indicates that for the teacher the use of *what's the same, what's different* was significant in terms of the development of her practice; the second provides insight into the nature of this relationship. Reference is made to thinking in the second quotation, perhaps the habit the teacher refers to is the habit of thinking in a particular way, a way that deepens knowledge. It would seem that the knowledge she refers to is subject knowledge rather than pedagogic knowledge, since it is common to both teachers and pupils. It would seem that she is claiming that not just her pupils, but own her subject knowledge develops through thinking in a particular way, prompted by the question.

Central to the construct of same and different is the idea of comparison. Comparison is important in mathematics and central to classification and defining of concepts. It also has the potential to make connections through the *things that are the same* and the things that connect are likely to be properties of the concept, thus developing and deepening conceptual understanding. The question is simple but potentially also profound *a few simple questions, like what's the same, what's different.....they've made the biggest difference... and children are opening their eyes to so much more* (Int4 20:51.7 - 20:53.7). It would seem that this question provided an important vehicle for the application of subject knowledge within teachers' practice.

Another key vehicle was the use of the construct of *mathematical powers*, taken from the work of Mason (2005). The powers are divided into four pairs: conjecturing and convincing; specialising and generalising; imagining and expressing; and finally organising and classifying. These were taken by teachers and made *child friendly* through turning them into *super heroes*, for example *Captain Conjecture*, who has the power to conjecture and convince. These were intended as ways of thinking in mathematics that pupils might develop awareness of and utilise in their learning of the subject. Teachers claim that they have been successful, *the introduction of mathematical powers through the super heroes, that whole idea of mathematical language has embedded much more thinking skills into the children in school and that has made a massive difference, not just for me, but all of the people within the school* (Int5 1:49.1 - 2:09.4). The powers as a vehicle for language which in turn develop, what I am interpreting as mathematical thinking is an interesting insight and is discussed in Chapter 2. The use of the mathematical powers provide a framework for talk, a teacher says *In my school the children come into nursery with a very low level of language and the big ideas and Mason's powers and the vocabulary have really helped the children reason and explain, where before they wouldn't* (Int5 2:39.6 - 3:16.2).

It is interesting how some powers seem to have had a greater impact than others as exemplified in the graph below. Conjecturing and convincing are by far the strongest, with very young children adopting the language. My hypothesis is that the use of the terms conjecturing and convincing lend themselves to talk and provide a framework within which mathematical talk can take place. Professional development programmes for teachers might promote the virtues of talk within mathematics classrooms, and it is highly probable that the teachers on the study had encountered such experiences, however what appeared to make the difference in the context of my programme is the presence of a vehicle for this to happen. This therefore is a key consideration for teacher professional development and for my own future practice.

Concluding Remarks

This chapter has analysed the findings from my study, developing theory which might be taken forward and in the context of practitioner research be applied to my practice. Key themes have emerged involving the nature of teacher subject knowledge, how it develops and its influence on practice. Teachers' subject knowledge when deep and relational in nature includes listening and representational knowledge and the ability to make connections and generalise mathematical concepts. The nature of the subject knowledge developed by the teachers is not what they had expected at the start of the programme, but as it developed it had a profound impact on their practice. Factors that supported its development include a focus on five big ideas that provided a connective structure within which to develop their understanding and make connections; engagement in and reflection on mathematics developed the way teachers thought about mathematics and this was also supported by academic reading; opportunities to network and develop alongside other teachers were also influencing factors in teacher development. The relationship between subject knowledge and pedagogic practice is strong and the two are demonstrated to be inextricably bound together and therefore the implication is that they should both be considered within professional development contexts. In particular vehicles to transfer professional development into practice are important. These are all learning points which I will apply to my future practice within the context of teacher professional development.

Chapter 6: The Conclusion

Having analysed and discussed the findings of my research this is the final chapter of my thesis. Its purpose is to:

- summarise the key findings from my research in relation to the research questions, highlighting in particular the new ideas which contribute to the existing research;
- identify the implications for my own practice and make recommendations to other professionals;
- make recommendations for future policy;
- make suggestions for further areas of research to enhance understanding.
- outline strategies for dissemination of my research

Key findings and new knowledge

The research addressed three questions in relation to the development of primary teachers subject knowledge in mathematics, these were:

- What is the nature of the subject knowledge developed by teachers engaged in the professional development programme?
- How does teacher knowledge develop and what factors have influenced its development?
- In what ways does the development of subject knowledge interact with and influence the practice of teachers?

The new knowledge regarding each of these questions will be discussed in turn.

The nature of teacher subject knowledge

Teachers within the study developed a particular type of subject knowledge which represented a changed way of thinking about mathematics. It is characterised by a connectionist form of

knowledge (Askew et. al., 1997) which is relational in nature (Skemp, 1976). The connectionist form of knowledge extends to seeing structures and relationships within concepts, and between concepts and seeing the *bigger picture* of mathematics as a connected network of ideas which includes mathematical processes, such as mathematical thinking, generalisation and the use of representational forms of mathematics to support conceptual learning.

The way that the teachers in the study think about mathematics has been transformed. They look for and expect to find connections in the mathematics; they use questioning, resources and representations of mathematics to promote this process with pupils. They also listen carefully to their pupils and engage in reasoning. They see being a mathematician, as not just about *knowing stuff* but as a way of thinking, the importance of thinking mathematically is extremely strong in the data and central to the nature of the teachers' subject knowledge.

There is a great deal written about the necessary nature of teacher subject knowledge and in recent years there has been a shift away from focusing only on content in terms of knowledge of facts, procedures and techniques. Many writers argue for a deeper form of knowledge, for example: Rowland et al (2009); laonne and Cockburn (2008); Lowenberg-Ball (2008); Askew et al (1997); Gray and Tall (1994) and Sfard (1991); and Skemp (1989), one in which making connections between areas of mathematics and generalising concepts are important. The teachers in my study appear to have developed this form of knowledge. The significance of my research is that this form of knowledge is being argued for, not by academics but by teachers involved in practice. The unique insight that these teachers provide in terms of the nature and development of their subject knowledge and its impact on their professional practice is significant and I believe is worth sharing with others through the dissemination of my research.

Recommendations for practice regarding the necessary nature of teacher subject knowledge

My recommendations for practice are that the development of primary teachers' subject knowledge should be of a deep, connected and relational form as described in the study and should include both listening and representational knowledge. As indicated by teachers in the research, this is the type of knowledge that they were lacking in and it is the form of knowledge their own findings found to be necessary for effective teaching of primary mathematics.

The development of teacher subject knowledge

My study was set within the context of a teacher professional development programme and it sought to develop new insights regarding professional development that I might apply to my practice. I have identified two key areas where new learning was developed that builds on current research:

- The development of teacher subject knowledge is dynamic and continuous and has the potential to develop within the context of practice.
- An integrated structure applied to the programme design had significant impact on teacher development

Each of these will be discussed in turn

Teacher subject knowledge continues to develop within the context of practice

The research in the area of how teacher subject knowledge develops is much less than that which seeks to define the necessary nature of teacher subject knowledge. From my study three new, but related ideas have emerged which add to the research literature. These are:

- that teacher subject knowledge can continue to develop within the act of teaching;
- the development of subject knowledge can be a two way process between teachers and pupils.

- the type of pedagogy applied within the classroom setting influences the development of teacher subject knowledge;

The development of listening knowledge, knowing what to listen for was significant in my research. Teachers are actively promoting and then listening for pupils' mathematical reasoning. This activity is providing the opportunity for engaging teachers in the process of mathematical reasoning as they seek to make sense of pupils' reasoning. The pedagogy that teachers have developed has resulted in a classroom climate of conjecturing, reasoning and looking for connections, where subject knowledge has the potential to develop for both teachers and pupils alike. Thus there are opportunities for the continued development of teacher subject knowledge.

The development of representation knowledge was significant in my study. The importance of this form of knowledge is already well attested in the research literature (Thwaites et al., 2005 Ma., 1999, Lesh et al., 1987). The development of teachers' representation knowledge became a strong feature within my study. It took the form of teachers making greater use of mathematical representations in their practice to develop pupil learning, as the Chinese teachers' in Ma's (1999) research did. My research adds to the research literature in the effective use of representations to develop teacher subject knowledge in terms of structural and relational understanding which then acted as tools to develop pupils' subject knowledge. This extended to teachers encouraging pupils to represent mathematics in their own way which then had the potential to provide a new way for teachers to see the mathematics. Thus the development of subject knowledge became a two way process between teachers and their pupils.

The design of the programme provided a connective framework that supported teacher professional development

The professional development programme that provided the context for the research was structured around five big ideas: mathematical thinking, pattern, representation, proportionality

and generalisation. These had a significant impact on teacher development as indicated in the data where the reported impact is extremely strong. The intention was that they would provide a connective framework in which the development of connective orientated teachers (Askew and Brown et al., 1997) might take place. This had an even greater impact than anticipated. The big ideas became permanent aspects of teachers' practice and defined the way they thought about mathematics. A teacher reports *big Ideas - made me think about the connections between all of the elements of maths we teach, moving the curriculum around to make the most of these connections.* (Q1 Q4 L7). An interview with a group of teachers one year after completion of the two year programme confirmed their sustainability. They were continuing to be the dominant force in informing lesson design and teaching. Previous research has identified aspects of these five ideas as effective within professional development programmes in particular the presence of mathematical thinking (Swan and Swain., 2010, Warren, 2009, Steinberg et al., 2004) but not the design of a development programme that is driven solely by a discrete set of big ideas, which are returned to again and again in an iterative manner. Professional development courses tend to be organised into discrete areas and the opportunity to make connections is not exploited, for example *The Five Day Course* (DfEE, 1999). Not only did the five big ideas form an umbrella structure in which all of the other ideas and development sat, but in the second year of the programme the five ideas began to merge and interact with each other, such was the propensity for teachers to make connections. Teachers began to see generalisation as a significant element within mathematical thinking and solving problems in mathematics (Int1 25:36.7 - 25:55.1). There was the recognition that pattern and its underlying structure led to generalisation (Int6 20:58.5 - 21:44.8). Representations exposed structure and pattern and communicated mathematical thinking (Q1 Q5 L55). After the first cohort I made the decision to merge the five big ideas in the second year of the programme rather than have one of them as a focus for each session, this was effective, as a teacher says *in terms of the structure of the course I think, this second year for me has been, the fact that we haven't broken it up into the five areas has worked really well, you've*

mixed them together a bit more and we have then been able to see the links (Int2 4:19.6 - 4:33.8).

However I would argue that exploring them separately in the first year of the programme enabled a deep understanding to be developed and learning embedded.

The five big ideas provided a coherent structure on which to hang the programme and develop the connective knowledge which is strongly represented within the data. Teachers' repeated reference to these ideas in the data indicates that they had a profound influence on their development. The ideas provided a structure within which to facilitate the making of connections in mathematics and their regular revisiting within the course provided a framework for deepening understanding and embedding in practice. Teachers who were interviewed one year after completion of the programme were still using the five big ideas to inform their planning and teaching. These were ideas that were deeply understood, firmly embedded and continuing to influence practice. They also provided a shared language in which to talk about and think about mathematics.

A second element of connectivity which emerged from the data was the integration of pedagogy with subject knowledge. *The fact that the programme included the development of subject knowledge alongside the pedagogy of teaching and learning maths* (Q1 Q7 L40) was significant to teachers' development. The connection between subject knowledge and pedagogy where one influences the other emerged strongly as a significant phenomenon.

A third element of connectivity is the integration of theory and academic literature within the context of doing mathematics and developing subject knowledge. Teachers engaged in mathematics, not in an isolated sense, but in the *light* of theory and research literature. Particular reference is made to Mason, Gattengo and Skemp. Also the interaction with research where there was a mathematical task presented in the research that teachers could engage with as learners of

mathematics (Miyakawa and Winslaw, 2009) provided a more experiential and deeper insight into the findings of the research.

Recommendations for practice in relation to the development of teacher subject knowledge

My recommendations for practice are to bear in mind the importance of representations in mathematics as extensions of teachers' subject knowledge, if teachers are unable to represent concepts in a variety of ways, demonstrating understanding of mathematical structures then their subject knowledge is insufficient. The use of representations both as a tool to develop teacher subject knowledge and as a tool for transference of this knowledge into practice will be significant in my practice. An important point however that emerged from my research was that it was changes in and the development of teacher subject knowledge that enabled teachers to use representations effectively to support learning. I will model good practice through the careful selection of representations that I use with teachers to develop their subject knowledge, and making the importance of their use in developing learning explicit.

The design of teacher professional development around a connective structure of just a few key ideas that are frequently returned to was effective within my programme. It facilitated the development of deep and sustainable learning that transfers into practice with a dynamic and continued impact on development and application to professional practice. My recommendation for practice is not that the structure of the five big ideas should necessarily be the same ideas. There should however be a structure around just a few key ideas to facilitate the iterative process of the development of deep and sustainable learning.

The fact that subject knowledge has the propensity to continue to develop within the context of practice is significant. This will require careful consideration in the design of professional

development to ensure that the right structures are developed to enable this to happen, in particular the development of teacher subject knowledge as a way of thinking. This will be key to my own practice and a recommendation for the practice of other leaders of teacher professional development.

The interrelationship of subject knowledge and professional practice

The interrelationship between the teachers' developed subject knowledge and their professional practice was interesting to observe and analyse and new learning emerged from the data; firstly in relation to the transfer of knowledge into practice and secondly the influence of subject knowledge on practice.

Transference of Professional Development into Practice

Transference of professional development into practice is arguably the most crucial outcome of professional development but also probably the hardest to achieve (Back et al., 2009, Anghileri, 2006, Houssart 2000). I have identified two key elements in my programme which supported the process and add to the research literature. These are the structure of the programme around five big ideas as discussed above and the use of particular constructs which became vehicles for transference of professional development into classroom practice. I have identified two particular vehicles, mention of which is strong within the data: the question *what's the same and what's different* and the application of *mathematical powers* (Mason et al., 1982) to teaching.

The question *what's the same and what's different* was adapted from Mason et al's work (1982) on developing mathematical thinking. Teachers were given the directed task very early in the programme of selecting two objects, pictures, numbers, symbols or mathematical expressions and asking pupils *what's the same, what's different* about them, at the start of each lesson for the period of a week. The purpose was to engage pupils in making comparisons and through the process make mathematical connections. Teachers report that the more pupils engaged in this

activity, the better they became at making connections in mathematics alongside development of mathematical reasoning. Reference to the construct is strong across the data and it became a regular feature of teaching, thus bringing the process of making connections into classroom practice.

Teachers took ownership of the construct of the four pairs of *mathematical powers*: conjecturing and convincing, specializing and generalizing, imagining and expressing, organising and classifying and they became *super heroes with super powers*, with names such as *Captain Conjecture*. The construct of conjecturing and convincing is particularly strong and acted as a vehicle for the development of reasoning and mathematical thinking. Thus acting as a vehicle for transference of a connectionist construct into practice and contributing to a classroom culture in which teachers' subject knowledge continued to develop.

The Influence of Subject Knowledge on Professional Practice

It is important in any professional context which involves the development of teachers' subject knowledge that the developed knowledge influences practice. There is significant evidence within the data that teachers have changed the way in which they teach. Subject knowledge and pedagogy are so interconnected within the data that it is difficult to distinguish between the two. Teachers' subject knowledge sits within the context of their teaching and their desire to improve their teaching is their motivation for the development of their subject knowledge. The newly acquired subject knowledge is as discussed; a way of thinking about mathematics where reasoning and relational understanding are important and connections are sought after and made. Believing that it is important for their pupils to think and reason about mathematics and connect new knowledge with prior knowledge naturally spills over into pedagogy and influences the way in which teachers teach.

Structures and constructs used within the course have supported the transference of knowledge into practice and an intertwining of subject knowledge and pedagogy has occurred, facilitated by vehicles such as those discussed above in terms of the question regarding same and different and the use of mathematical powers (Mason et al., 1982). The continuance of the development of teachers' subject knowledge within the context of their practice also represents a significant relationship between subject knowledge and pedagogy. Teachers' practice was transformed and they became connectionist orientated teachers (Askew and Brown et al., 1997) where making connections in mathematics supported the development of not only pupils' subject knowledge, but their own continued development of subject knowledge within the context of practice. The nature of the pedagogy that developed facilitated the continuation of development of subject knowledge. This sheds new light on the development of teachers' subject knowledge. If teachers' subject knowledge is seen as a way of thinking, something that is dynamic and that is deepened over time, then such knowledge can continue to develop outside of any professional development programme and indeed with the context of practice.

Recommendations for practice

My recommendations for future practice are to carefully consider the use of vehicles for transference of professional development into practice. Transference does not happen automatically and needs to be considered within the planning of teacher professional development. The use of the construct of mathematical powers as a vehicle, in particular the power to conjecture is embedded within the reasoning strand of the New Curriculum with the requirement that pupils should *reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language* (2013). The vehicle of conjecturing and convincing could be a valuable one for meeting the requirements of the National Curriculum.

How the findings of my study fits into the current educational context

Within the context of the 2014 National Curriculum for mathematics (DfE, 2013) the demands on teacher subject knowledge are significant. For example there is the requirement to calculate with fractions in all four operations and introduce formal algebra. Teachers will need to know *more* mathematics, but their knowledge of mathematics also needs to be deeper. This is particularly necessary to address the requirements of the three aims of the curriculum in the form of fluency, mathematical reasoning and problem solving. All three of these aims require both the nature and depth of mathematical knowledge developed by the teachers in my study. Fluency is intended to move beyond knowledge of facts and procedures and requires the ability to make connections, to move between different representations of mathematics and recognise the same mathematics in different contexts. Reasoning, like fluency requires the ability to make connections and think mathematically. The findings by Nunes et al (2009) are startling; if children are not reasoning in mathematics by the time they reach Year 4 then they are unlikely to be successful in the subject. It seems indisputable that reasoning pupils require teaching by reasoning teachers. Finally problem solving; pupils will only be able to solve problems in mathematics if their knowledge of mathematics includes all three types of knowledge, knowing that, knowing how and knowing why. The third of these, the *knowing why*, mathematical procedures and techniques work and how and why concepts connect with each other was originally weak for most of the teachers in my study. This knowledge developed through the programme. Problems in mathematics are too diverse to memorise strategies and techniques to address them, deep conceptual knowledge and the ability to generalise mathematics is required to recognise and apply the mathematics in unfamiliar contexts. To develop this form of knowledge and skills pupils need teachers with that knowledge and those skills. Long gone are the days when rote learning of mathematics might have been sufficient. The demands of the 21st Century require the ability to reason and problem solve, applying mathematics flexibly and creatively to a wide variety of contexts (Richland et al.,

2012). Within my study a group of teachers describe what they have identified that teachers need to know. These included:

- understanding the mathematics, when probed what they meant by this, they talked about seeing and understanding why it worked, making reference to relational understanding (Int3 12:33.0 - 13:25.7);
- the use of representations to help children to develop a deep understanding of mathematics (Int3 15:16.5 - 15:30.7);
- the need for flexibility and being able to adapt, depending on what comes up in the lesson (Int3 15:54.2 - 16:19.0), equating that to a deeper level of understanding than many teachers currently have.

The demands of the 2014 New Curriculum (DfE, 2013) require a deep form of subject knowledge, the nature of which is expressed in the findings of my study. The findings of my study demonstrates not only the nature of knowledge necessary for effective teaching, but also how this form of knowledge is developed and factors which support its development.

Looking Forward: Recommendations for Policy

My study has focused on the development of primary teachers' subject knowledge. Deficiencies in teacher subject knowledge are widely recognised, as discussed in Chapter 2. One strategy to address this has been the voluntary raising of entry qualifications by institutions where the common requirement has shifted from the statutory requirement of a Grade C to a grade B at GCSE. This may not however represent the depth of knowledge that teachers require for effective teaching of primary mathematics. The national MaST programme was an attempt to develop such depth of knowledge but funding has recently ceased and so the opportunity is no longer available for most teachers.

At present there is no agreed national framework which outlines the required subject knowledge for the teaching of primary mathematics. I was recently involved in the writing of a framework for mathematics specialist programmes within Initial Teacher Training (ITT), but this is guidance only and not a requirement. Until such a framework exists there will be no consistency in the development of teacher subject knowledge in mathematics and many teachers will not receive the professional development they require for effective teaching of the subject.

In my role as Director for Primary Mathematics (NCETM) I have begun to argue for a National Framework which outlines both the content and nature of the subject knowledge required by primary teachers for effective teaching of mathematics. In addition to this I have recommended to the ACME policy group for initial teacher training a bespoke qualification for teaching mathematics which develops deep and connective knowledge of fundamental concepts in mathematics, reflecting the knowledge that is possessed by the Chinese teachers in Ma's study (Ma, 1999) and the nature of the knowledge developed by the teachers in my study.

Looking Forward: Recommendations for Future Research

I recommend three key areas for future research. The first is in the area of teacher professional development and the development of teacher subject knowledge. As indicated in Chapter 2, there is much less research in this area than there is in the exploration of the necessary nature teacher subject knowledge. It is one thing to know the nature of the knowledge required to teach, but of equal importance is understanding how this knowledge is developed and effective strategies for doing so. My research has suggested some strategies and although they indicated positive impact on teacher development they were set in a particular context. However the study represents a significant sample of participants and I believe there is an argument for them being generalisable. The difficulty however might be that they were developed over the period of two years, a time

period well in excess of most teacher professional development . Research is required to see whether or not they are effective over a shorter time period.

My second recommendation relates to the structure of professional development. My study identified that structuring professional development around five big ideas in mathematics provided the opportunity for the ideas to be regularly revisited in different forms, developing depth of understanding. These ideas became embedded and central to the way teachers now think about mathematics and they continue to impact on their practice. Thus they were effective in achieving sustained teacher development that continues to develop within the context of practice. I am currently leading a national programme where the development of deep mathematics subject knowledge of primary teachers is central. The teachers involved in this programme are also to become leaders of other teachers' professional development. I have structured the programme around five big ideas and am making it explicit how these are a vehicle for professional development. The five big ideas selected for this programme are not identical but do overlap with the ideas used within my study. It would be valuable to research their impact on teacher development, in particular how they support the teachers in developing deep subject knowledge, enable them to make connections and how they impact on teachers' professional practice in a sustained manner.

My third recommendation relates to the need for a nationally agreed framework for the development of primary teacher subject knowledge, laying out clearly the required knowledge recommendations for development. Such a framework would require researching, trialing and evaluation. This I believe has the potential to make a significant difference to the teaching and learning of mathematics in primary schools.

Looking Forward: Dissemination of my research

I have made several recommendations for future practice, referenced to in the preceding sections. These are both recommendations for myself and others involved in the professional development of teachers. I have also made recommendations for future policy. There are several avenues open to me to facilitate the process of dissemination in addition to the publication of research papers. As indicated above my national role provides access to both teachers and policy makers. I have recently been involved in the writing of national materials to support assessment of mathematics. Within the materials I have included the character of *Captain Conjecture* as a vehicle to promote reasoning and mathematical thinking within primary classrooms and written a magazine article on its benefits.

I have recently been involved in the development of national recommendations for initial teacher training regarding the mathematics development of primary trainee teachers. The findings from my research regarding the development of teacher subject knowledge are informing the development of work in this area; including the recognition that teachers require a deep and connective form of subject knowledge. As with the teachers the ideas developed, have become embedded and part of my own thinking. I have also been invited to speak at the International Conference of Mathematics Education (ICME) this year; once again this will provide an opportunity to disseminate some of the findings of my research.

I am sure that the opportunities to share what I have learnt will continue to arise through my professional role as a leader in the development of primary mathematics and my work with teachers as a developer of those who lead professional development.

References

- ADLER, J. & DAVIS, Z. (2011) Modelling Teaching in Mathematics Teacher Education and the Constitution of Mathematics for Teaching, in ROWLAND, T. & RUTHVEN, K. (Eds.), *Mathematical Knowledge in Teaching*. Heidelberg, Springer.
- AGAR, M. (1986) *Speaking of Ethnography, Qualitative Research Methods Series 2.*, Beverley Hills, Sage.
- AINLEY, J., & LUNTLEY, M. (2007) Towards an articulation of expert classroom practice. *Teaching and Teacher Education*, 23, no7 pp 1127-1138.
- AKERLIND, G. S. (2012). Variation and commonality in phenomenographic research methods. *Higher Education Research & Development*, vol 31(1), pp. 115-127.
- ALEXANDER, R., ROSE, J. & WOODHEAD, C. (1992) *Curriculum Organisation and Classroom Practice in Primary Schools: A Discussion Paper*. London, Department of Education and Science (DES).
- ANGHILERI, J. (2006) A Study of the Impact of Reform on Students' Written Calculation Methods after Five Years' Implementation of the National Numeracy Strategy in England, *Oxford Review of Education*, vol. 32, pp. 363-380.
- ASKEW, M., BROWN, M., RHODES, V., JOHNSON, D. & WILIAM, D. (1997) *Effective Teachers of Numeracy*. London, Kings College.
- BACK, J., HIRST, C., DE GEEST, E., JOUBERT, M. & SUTERLAND, R. (2009) *Final Report: Researching Effective CPD in Mathematics Education (RECME)*. London, Department for Children, Schools and Families (DCSF).
- BAILEY, K. D. (1994) *Methods of Social Research*. New York, NY, Free Press.
- BALL, D. L. (1990a) Breaking with Experience in Learning to Teach Mathematics: the Role of a Pre-service Methods Course, *For the Learning of Mathematics, an International Journal of Mathematics Education*, vol. 10, pp. 10-16.

- BALL, D. L. (1990b) The Mathematical Understandings That Prospective Teachers Bring to Teacher Education, *The Elementary School Journal*, vol. 90 (4), pp. 449-466.
- BALL, D. L. (1990c) Prospective Elementary and Secondary Teachers' Understanding of Division, *Journal for Research in Mathematics Education*, vol. 21, pp. 132-144.
- BALL, D. L. (1993) With an Eye on the Mathematical Horizon: Dilemmas of Teaching Elementary School Mathematics, *The Elementary School Journal*, vol. 93 (4), pp. 373-397.
- BALL, D. L., SLEEP, L., BOERST, T. A. & BASS, H. (2009) Combining the Development of Practice and the Practice of Development in Teacher Education, *The Elementary School Journal*, vol. 109 (5), pp. 458-474.
- BALL, D. L., THAMES, M. H. & PHELPS, G. (2008) Content Knowledge for Teaching: What Makes it Special?, *Journal of Teacher Education*, vol. 59, pp. 389-407.
- BARMBY, P., BILSBOROUGH, L., HARRIES, T. & HIGGINS, S. (2009) *Primary Mathematics: Teaching for Understanding*. Milton Keynes, Open University Press.
- BAROODY, A. J. (2003) The Development of Adaptive Expertise and Flexibility: The Integration of Conceptual and Procedural Knowledge, in BAROODY, A. J. & DOWKER, A. (Eds), *The Development of Arithmetic Concepts and Skills: Constructing Adaptive Expertise*. Mahwah, NJ, Lawrence Erlbaum Associates Publishers. pp. 1-34.
- BAUGH, J. B., SABER HALLCOM, A. & HARRIS, M. E. (2010) Computer Assisted Qualitative Data Analysis Software, *Revista Digital del Instituto Internacional de Costos*, vol. 6, pp. 69-81.
- BECKMANN, S. (2004) Solving Algebra and Other Story Problems with Simple Diagrams: A Method Demonstrated in Grade 4-6 Texts Used in Singapore, *Mathematics Educator*, vol. 14, pp. 42-46.
- BILLS, C. & GRAY, E. (1999) Pupils' Images of Teachers' Representations, *Proceedings of the Conference of the International Group for the Psychology of Mathematics Education (PME)*, vol. 23 (2), pp. 113-120
- BOATENG, W. (2012) Evaluating the Efficacy of Focus Group Discussion (FGD) in Qualitative Social Research, *International Journal of Business & Social Science*, vol. 3, pp. 54-57.

- BOYLAN, M. (2010) 'It's Getting me Thinking and I'm an Old Cynic': Exploring the Relational Dynamics of Mathematics Teacher Change, *Journal of Mathematics Teacher Education*, vol. 13, pp. 383-395.
- BRITISH SOCIETY FOR THE HISTORY OF MATHEMATICS. & PÓLYA, G. (1954) *Mathematics and Plausible Reasoning*, Princeton, NJ, Princeton University Press.
- BROWN, L. & COLES, A. (2010) Mathematics Teacher and Mathematics Teacher Educator Change and Insight through Theoretical Perspectives, *Journal of Mathematics Teacher Education*, vol. 13, pp. 375-382.
- BRUNER (1966) *Toward a theory of instruction* (Vol. 59). Harvard University Press.
- BURGESS, H., SIEMINSKI, S. & ARTHUR, L. (2006) *Achieving Your Doctorate in Education*. London, Sage Publications Ltd.
- BURTON, L. (2002) Methodology and Methods in Mathematics Education Research: Where is "The Why"? in GOODCHILD, S. & ENGLISH, L. (Eds.) *Researching Mathematics Classrooms, A Critical Examination of Methodology*. London, Praeger Publishers.
- CARPENTER, T. P., FENNEMA, E., PETERSON, P.L., & CAREY, D.A. (1988) Teachers' Pedagogical Content Knowledge of Students' Problem Solving in Elementary Arithmetic, *Journal for Research in Mathematics Education*, vol. 19, pp. 385-401.
- CARPENTER, T. P., FRANKE, M. L. & LEVI, L. (2003) *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School*. Portsmouth, NH, Heinemann.
- CATTERALL, M. & MACLARAN, P. (1998) Using Computer Software for the Analysis of Qualitative Market Research Data, *Journal of Market Research Society*, vol. 40, pp. 207-222.
- CHARMAZ, K. (2003) Grounded theory, *Strategies of Qualitative Inquiry*, vol. 2, p. 249.
- CLARKE, D. & HOLLINGSWORTH, H. (2002) Elaborating a Model of Teacher Professional Growth, *Teaching and Teacher Education*, vol. 18, pp. 947-967.
- COCHRAN-SMITH, M. & LYTLE, S. L. (1993) *Inside/Outside: Teacher Research and Knowledge*. New York, NY, Teachers College Press.

- COCHRAN SMITH, M., & LYTLE, S. L. (2009). *Inquiry as stance: Practitioner research for the next generation*. Teachers College Press.
- COHEN, L., MANION, L. & MORRISON, K. (2007) *Research Methods in Education*. London, Routledge Ltd.
- CORBIN, J. & STRAUSS, A. (2008) *Basics of Qualitative Research, 3rd Edition*. Thousand Oaks, CA, Sage.
- CORBIN, J. & STRAUSS, A. (2014) *Basics of Qualitative Research: Techniques and Procedures for Developing Grounded Theory*. Thousand Oaks, CA, Sage publications.
- COUSIN, G. (2009) *Researching Learning in Higher Education, An Introduction to Contemporary Methods and Approaches*. London, Routledge.
- CRESWELL, J. W. (2012) *Educational Research: Planning, Conducting, and Evaluating Quantitative and Qualitative Research*. Boston, MA, Pearson Education Inc.
- CUISENAIRE, G. & GATTEGNO, C. (1955) *Numbers in Colour*. London, Heinemann.
- DARLING-HAMMOND, L. & MCLAUGHLIN, M. W. (1995) Policies that Support Professional Development in an Era of Reform, *Phi Delta Kappan*, vol. 76, pp. 597-604.
- DAVIS, B. (1997) Listening for Differences: An Evolving Conception of Mathematics Teaching, *Journal for Research in Mathematics Education*, vol. 28, pp. 355-376.
- DAVIS, B. & SIMMT, E. (2006) Mathematics-for-Teaching: An Ongoing Investigation of the Mathematics that Teachers (Need to) Know, *Educational Studies in Mathematics*, vol. 61, pp. 293-319.
- DE GEEST, E. (2011) Roles of Research Utilization in Professional Development of Mathematics Teachers, *Research in Mathematics Education*, vol 13(1) pp. 77-78.
- DE LA CINTA MUNOZ-CATALAN, M., CARNILLO YANEZ, J. & CLIMENT RODRIGUEZ, N. (2010) Mathematics Teacher Change in a Collaborative Environment: To What Extent? and How?, *Journal of Mathematics Teacher Education*, vol. 13, pp. 425-439.
- DFE (2012) *Teachers Standards*. London, Department for Education.

- DFE (2013) *The National Curriculum in England: Framework Document*. London, Department for Education.
- DFE (2014) *Initial teacher training criteria supporting advice Information for accredited initial teacher training providers*. London, Department for Education.
- DFEE (1997) *Teaching: High Status, High Standards, Requirements for Courses of Initial Teaching Training*. London, Department for Education and Employment.
- DFEE (1999) *The National Curriculum Handbook for Primary Teachers in England*, London Department for Education and Employment.
- DFES (2004) *The National Curriculum Handbook for Secondary Teachers in England*, London Department for Education and Skills.
- DFEE (1999) *The Five Day Course*. London, Department for Education & Employment.
- EHRICH, L. C. (2003) Phenomenology: The Quest for Meaning, Chapter 4 in O'Donoghue, T. & Punch, K. (Eds.) *Qualitative Educational Research in Action: Doing and Reflecting*. Abington, Routledge.
- ERLANDSON, D. A. (1993) *Doing Naturalistic Inquiry: A Guide to Methods*. London, Sage.
- ERNEST, P. (1999) Forms of Knowledge in Mathematics and Mathematics Education: Philosophical and Rhetorical Perspectives, *Educational Studies in Mathematics*, vol. 38, pp. 67-83.
- FOX, M., GREEN, G. & MARTIN, P. (2007) *Doing Practitioner Research*. London, Sage.
- FRANKE, M. L., CARPENTER, T. P., LEVI, L. & FENNEMA, E. (2001) Capturing Teachers' Generative Change: A Follow-up Study of Professional Development in Mathematics, *American Educational Research Journal*, vol. 38, p. 653.
- FULLAN, M. (2001a) *Leading in a Culture of Change*. San Francisco, California, Jossey-Bass.
- FULLAN, M. (2001b) *The New Meaning of Educational Change*. New York, NY, Teachers College Press.
- FULLAN, M. & EARL, L. (2002) United Kingdom National Literacy and Numeracy Strategies: A Large Scale Reform, *Journal of Educational Change*, vol. 3, pp. 1-5.

- GARET, M. S., PORTER, A. C., DESIMONE, L., BIRMAN, B. F. & YOON, K. S. (2001) What Makes Professional Development Effective? Results from a National Sample of Teachers, *American Educational Research Journal*, vol. 38, pp. 915.
- GATES, P. (Ed.) (2001) *Issues in Mathematics Teaching*. London, Routledge Falmer.
- GIAQUINTO, M. (2011) *Visual Thinking in Mathematics*. Oxford, Oxford University Press.
- GILES, T., KING, L. & DE LACEY, S. (2013) The Timing of the Literature Review in Grounded Theory Research: an Open Mind Versus an Empty Head, *Advances in Nursing Science*, Vol. 36, pp. E29-E40.
- GLASER, B., & STRAUSS, A (1967) *The Discovery of Grounded Theory: Strategies for Qualitative Research*. London, Wiedenfeld and Nicholson.
- GOETZ, J. P. & LECOMPTE, M. D. (1984) *Ethnography and Qualitative Design in Educational Research*. Orlando, FL, Academic Press.
- GOOS, M. & GEIGER, V. (2010) Theoretical Perspectives on Mathematics Teacher Change, *Journal of Mathematics Teacher Education*, vol. 13, pp. 499-507.
- GOULDING, M., ROWLAND, T. & BARBER, P. (2002) Does it Matter? Primary Teacher Trainees' Subject Knowledge in Mathematics, *British Educational Research Journal*, vol. 28, pp. 689-704.
- GOUTARD, M. (1964) *Mathematics and Children: a Reappraisal of our Attitude*. Reading, Educational Explorers.
- GRANEHEIM, U. H. L. & LUNDMAN, B. (2004) Qualitative Content Analysis in Nursing Research: Concepts, Procedures and Measures to Achieve Trustworthiness, *Nurse Education Today*, vol. 24, pp. 105-112.
- GRAY, E., PINTO, M., PITTA, D. & TALL, D. (1999) Knowledge Construction and Diverging Thinking in Elementary & Advanced Mathematics, *Forms of Mathematical Knowledge*, vol. 38, pp. 111-133.
- GRAY, E. M. & TALL, D. O. (1994) Duality, Ambiguity, and Flexibility: A "Proceptual" View of Simple Arithmetic, *Journal for Research in Mathematics Education*, vol. 25, pp. 116-140.

- GU, L., HUANG, R., & MARTON, F. (2004). Teaching with variation: A Chinese way of promoting effective Mathematics learning, in FAN, L. (ed.), *How Chinese learn mathematics: perspectives from insiders*, New Jersey; London: World Scientific. p309
- GUBA, E. G. & LINCOLN, Y. S. (1981) *Effective Evaluation: Improving the Usefulness of Evaluation Results Through Responsive and Naturalistic Approaches*. San Francisco, CA, Jossey-Bass.
- GUSKEY, T. R. (1986) Staff Development and the Process of Teacher Change, *Educational Researcher*, vol. 15, pp. 5-12.
- GUSKEY, T. R. (2002) Professional Development and Teacher Change, *Teachers and Teaching: Theory and Practice*, vol. 8, pp. 381-391.
- HARGREAVES, D.H. (1996) Teaching as a research-based profession: Possibilities and prospects, *Teacher Training Agency Annual Lecture*. London: Teacher Training Agency.
- HEATH, H., & COWLEY, S. (2004) Developing a grounded theory approach: a comparison of Glaser and Strauss. *International journal of nursing studies*, vol. 41(2), pp 141-150.
- HEGARTY, S. (2000) Teaching as a Knowledge-Based Activity, *Oxford Review of Education*, Vol. 26, pp. 451-465.
- HILL, H. & BALL, D. L. (2009) The Curious—and Crucial—Case of Mathematical Knowledge for Teaching, *Phi Delta Kappan*, vol. 91, pp. 68-71.
- HILL, H. C., ROWAN, B. & BALL, D. L. (2005) Effects of Teachers' Mathematical Knowledge for Teaching on Student Achievement, *American Educational Research Journal*, vol. 42, pp. 371-406.
- HOUSSART, J. (2000) The Role of Number Resources in the Daily Mathematics Lesson, *Proceedings of the British Society for Research into Learning Mathematics British Society for Research into Learning Mathematics (BSRLM)*, vol. 20(3), pp. 37-42
- HUTCHISON, A. J., JOHNSTON, L. H. & BRECKON, J. D. (2010) Using QSR-NVivo to Facilitate the Development of a Grounded Theory Project: an Account of a Worked Example, *International Journal of Social Research Methodology*, Vol. 13, pp. 283-302.

- IANNONE, P. & COCKBURN, A. D. (2008) If you can Count to Ten you can count to Infinity Really: Fostering Conceptual Mathematical Thinking in the First Year of Primary School, *Research in Mathematics Education*, vol. 10, pp. 37-51.
- IZSAK, A. (2008) Mathematical Knowledge for Teaching Fraction Multiplication, *Cognition and Instruction*, vol. 26, pp. 95-143.
- JOHNSON, R. B. & ONWUEGBUZIE, A. J. (2004) Mixed Methods Research: A Research Paradigm Whose Time Has Come, *Educational Researcher*, vol. 33, pp. 14-26.
- JOHNSTON-WILDER, S. & MASON, J. (2005) *Developing thinking in geometry*. London, Sage Publications Ltd.
- KASTBERG, S. E., D'AMBROSIO, B. & LYNCH-DAVIS, K. (2012) Understanding Proportional Reasoning for Teaching, *Australian Mathematics Teacher*, vol. 68, pp. 32-40.
- KITWOOD, T. M. (1997) Values in Adolescent Life: Towards a Critical Description. (Ph.D. Thesis) School of Education, University of Bradford.
- LEININGER, M. M. (1985) *Qualitative Research Methods in Nursing*. New York, NY, Grune & Stratton.
- LESH, R. A., HAMILTON, E. & KAPUT, J. J. (2007) *Foundations for the Future in Mathematics Education*. Mahwah, NJ, Lawrence Erlbaum Associates.
- LESH, R. & LANDAU, M. (1983) *Acquisition of Mathematics Concepts and Processes*. London, Academic Press.
- LESH, R., POST, T. & BEHR, M. (1987) Representations and Translations Among Representations in Mathematics Learning and Problem Solving, in LESH, R., POST, T. & BEHR, M., *Problems of Representation in the Teaching and Learning of Mathematics*. Hillsdale, NJ, Lawrence Erlbaum. pp. 33-40.
- LEWIS, C., PERRY, R. & MURATA, A. (2006) How Should Research Contribute to Instructional Improvement? The Case of Lesson Study, *Educational Researcher*, vol. 35, pp. 3-14.

- LILJEDAHN, P. (2010) Noticing Rapid and Profound Mathematics Teacher Change, *Journal of Mathematics Teacher Education*, vol. 13, pp. 411-423.
- LODICO, M. G., SPAULDING, D. T. & VOEGTLE, K. H. (2010) *Methods in Educational Research: From Theory to Practice*. San Francisco, CA, Jossey-Bass.
- LOEWENBERG BALL, D., THAMES, M. H. & PHELPS, G. (2008) Content Knowledge for Teaching, *Journal of teacher education*, vol. 59, p. 389.
- LOUCKS-HORSLEY, S., LOVE, N. & STILES, K. E. (2003) *Designing Professional Development for Teachers of Science and Mathematics*. Thousand Oaks, CA, Corwin Press.
- MA, L. (1999) *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States*. Mahwah, NJ, Lawrence Erlbaum Associates.
- MARTON, F. (1981). Phenomenography: Describing conceptions of the world around us. *Instructional Science*, vol. 10, pp. 177–200.
- MASON, J. (1994) Researching From the Inside in Mathematics Education. *Proceedings of the Eighteenth International Conference for the Psychology of Mathematics Education (PME 18)*, pp. 269–283.
- MASON, J. (2002) *Researching Your Own Practice: The Discipline of Noticing*. London, Routledge.
- MASON, J., BURTON, L. & STACEY, K. (1982) *Thinking Mathematically*. London, Addison-Wesley.
- MASON, J. & JOHNSTON-WILDER, S. (2004) *Fundamental Constructs in Mathematics Education*. London, Routledge Ltd.
- MASON, J., JOHNSTON-WILDER, S. & GRAHAM, A. (2005) *Developing Thinking in Algebra*. London, Sage (Paul Chapman).
- MASON, J. & SPENCE, M. (1999) Beyond Mere Knowledge of Mathematics: The Importance of Knowing-to Act in the Moment, *Educational Studies in Mathematics*, vol. 38, pp. 135-161.
- MASON, J., STEPHENS, M. & WATSON, A. (2009) Appreciating Mathematical Structure for All, *Mathematics Education Research Journal*, vol. 21, pp. 10-32.

- MCLAFFERTY, I. (2004). Focus group interviews as a data collecting strategy. *Journal of advanced nursing*, Vol 48(2), pp 187-194.
- MCNEIL, N. & JARVIN, L. (2007) When Theories Don't Add Up: Disentangling the Manipulatives Debate, *Theory into Practice*, vol. 46, pp. 309-316.
- MCNEIL, N. M. & UTTAL, D. H. (2009) Rethinking the Use of Concrete Materials in Learning: Perspectives From Development and Education, *Child Development Perspectives*, vol. 3, pp. 137-139.
- MEAD, G. H. (2009) *Mind, Self, and Society: From the Standpoint of a Social Behaviorist*. Chicago, IL, University of Chicago Press.
- MERRIAM, S. (1995) What Can You Tell From An N of 1?: Issues of Validity and Reliability in Qualitative Research, *PAACE Journal of Lifelong Learning*, vol. 4, pp. 50-60.
- MISHLER (1979). Meaning in context: Is there any other kind?. *Harvard Educational Review*, vol. 49(1), pp. 1-19.
- MIYAKAWA, T. & WINSLOW, C. (2009) Didactical Designs for Students Proportional Reasoning: an Open Approach Lesson and a Fundamental Situation, *Educational Studies in Mathematics*, vol. 72, pp. 199-218.
- MONTESORI, M. (1964) *The Montessori Method*. New York, Schocken.
- MORSE, J.M. (2015) Critical Analysis of Strategies for Determining Rigor in Qualitative Inquiry, *Qualitative Health Research* Vol. 25(9), pp.1212-1222
- MORSE, J. M., BARRETT, M., MAYAN, M., OLSON, K. & SPIERS, J. (2008) Verification Strategies for Establishing Reliability and Validity in Qualitative Research, *International Journal of Qualitative Methods*, vol. 1, pp. 13-22.
- MOYER, P. S. (2001) Are We Having Fun Yet? How Teachers Use Manipulatives to Teach Mathematics, *Educational Studies in Mathematics*, vol. 47, pp. 175-197.
- MURATA, A. (2008) Mathematics Teaching and Learning as a Mediating Process: The Case of Tape Diagrams, *Mathematical Thinking and Learning*, vol. 10, pp. 374-406.

NCTL (2015) *Initial teacher training criteria supporting advice*. Nottingham, National College for Teaching and Leadership

NOH, J., & WEBB, M. (2015). *Teacher Learning of Subject Matter Knowledge Through an Educative Curriculum. The Journal of Educational Research*, vol 108(4), pp. 292-305.

NUNES, T., BRYANT, P., HALLETT, D., BELL, D. & EVANS, D. (2009a) Teaching Children About the Inverse Relation Between Addition and Subtraction, *Mathematical Thinking and Learning*, vol. 11, pp. 61-78.

NUNES, T., BRYANT, P., SYLVA, K. & BARROS, R. (2009b) *Development of Maths Capabilities and Confidence in Primary School (Research Report DCSF-RR118)*. London: Department for Children, Schools and Families (DCSF).

NUNES, T., BRYANT, P. & WATSON, A. (2009c) *Key Understandings in Mathematics Learning*. London, Nuffield Foundation.

OFSTED (2008) *Mathematics: Understanding the Score*. London, Office for Standards in Education.

OFSTED (2013) *Distance Learning on Ofsted's Mathematics Reports*. London, Office for Standards in Education.

OLMEZ, I. B. (2014) Pre-service Teachers' Understandings of Division and Proportional Relationships with Quantities. (Unpublished Thesis) University of Georgia Theses and Dissertations -http://purl.galileo.usg.edu/uga_etd/olmez_ibrahim_b_201405_ma .

ORRILL, C. H. & BROWN, R. E. (2012) Making Sense of Double Number Lines in Professional Development: Exploring Teachers' Understandings of Proportional Relationships, *Journal of Mathematics Teacher Education*, vol. 15, pp. 381-403.

PAPE, S. J. & TCHOSHANOV, M. A. (2001) The Role of Representation(s) in Developing Mathematical Understanding, *Theory Into Practice*, vol. 40, pp. 118.

- PEHKONEN, E. & TURNER, G. N. (1999) Teachers' Professional Development: What Are the Key Change Factors for Mathematics Teachers?, *European Journal of Teacher Education*, vol. 22, pp. 259-275.
- PIAGET, J. (2013, originally published 1941) *Child's Conception of Number: Selected Works*. Abingdon, Routledge.
- PÓLYA, G. (1968) *Mathematics and Plausible Reasoning: Patterns of Plausible Inference*. Princeton, NY, Princeton University Press.
- POLYA, G., KADDOUCH, R., RENOU, M., COMTOIS, M. & DUBOIS, M.-J. (1957) *How to Solve it: a New Aspect of Mathematical Method*. Princeton, NJ, Princeton University.
- Milton Keynes, Open University Press.
- POUW, W., GOG, T. & PAAS, F. (2014) An Embedded and Embodied Cognition Review of Instructional Manipulatives, *Educational Psychology Review*, vol. 26, pp. 51-72.
- PRECIADO-BABB, A. P. & LILJEDAHN, P. (2012) Three Cases of Teachers Collaborative Design: Perspectives From Those Involved, *Canadian Journal of Science, Mathematics & Technology Education*, vol. 12, pp. 22-35.
- QSR (2010) *NVivo 9*. QSR International Pty Ltd.
- RESNICK, L. B. (1991) Shared Cognition: Thinking as Social Practice, *Perspectives in Socially Shared Cognition*, Washington, DC, American Psychological Association.
- RICHLAND, L. E., STIGLER, J. W. & HOLYOAK, K. J. (2012) Teaching the Conceptual Structure of Mathematics, *Educational Psychologist*, vol. 47, pp. 189-203.
- RITTLE-JOHNSON, B., KALCHMAN, M., CZARNOCHA, B. & BAKER, W. (2002) An Integrated Approach to the Procedural/Conceptual Debate, *Proceedings of 24th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education Conference*, p. 158. Discussion group PME-NA XXIV.

- RITTLE-JOHNSON, B., SIEGLER, R. S. & ALIBALI, M. W. (2001) Developing Conceptual Understanding and Procedural Skill in Mathematics: An Iterative Process, *Journal of Educational Psychology*, Vol. 93, pp. 346-362.
- ROSENBAUM, H. & SHACHAF, P. (2010) A Structuration Approach to Online Communities of Practice: The Case of Q&A Communities, *Journal of the American Society for Information Science and Technology*, vol. 61, pp. 1933-1944.
- ROWLAND, T., THWAITES, A., HUCKSTEP, P. & TURNER, F. (2009) *Developing Primary Mathematics Teaching: Reflecting on Practice with the Knowledge Quartet*. London, Sage Publications Ltd.
- RYKEN, A. E. (2009) Multiple Representations as Sites for Teacher Reflection about Mathematics Learning, *Journal of Mathematics Teacher Education*, vol. 12, pp. 347-364.
- SCHNEIDER, M., RITTLE-JOHNSON, B. & STAR, J. R. (2011) Relations Among Conceptual Knowledge, Procedural Knowledge, and Procedural Flexibility in Two Samples Differing in Prior Knowledge, *Developmental Psychology*, vol. 47, pp. 1525-1538.
- SCHOENFELD, A. H. (1992) Learning to Think Mathematically: Problem Solving, Metacognition, and Sense Making in Mathematics, in GROUWS, D., (Ed.), *Handbook of Research on Mathematics Teaching and Learning*, New York, NY, Macmillan. pp. 334-370.
- SCHON, D. A. (1983) *The Reflective Practitioner: How Professionals Think in Action*. New York, NY, Basic Books.
- SCHWAB, J. J. (1982) *Science, Curriculum, and Liberal Education: Selected Essays*. Chicago, University of Chicago Press.
- SCHWANDT, T. A. (2007) *The Sage Dictionary of Qualitative Enquiry*. Thousand Oaks, CA, Sage.
- SFARD, A. (1991) On the Dual Nature of Mathematical Conceptions: Reflections on Processes and Objects as Different Sides of the Same Coin, *Educational Studies in Mathematics*, Vol. 22, pp. 1-36.
- SFARD, A. (2008) *Thinking as Communicating: Human Development, the Growth of Discourses, and Mathematizing*. Cambridge, Cambridge University Press.

- SHE, X., LAN, W. Y. & WILHLEM, J. (2011) A Comparative Study on Pedagogical Content Knowledge of Mathematics Teachers in China and the United States, *New Waves - Educational Research & Development*, vol. 14, pp. 35.
- SHERIN, M. G. (2002) When Teaching Becomes Learning, *Cognition and Instruction*, vol. 20, pp. 119-150.
- SHULMAN, L. S. (1986) Those Who Understand: Knowledge Growth in Teaching, *Educational Researcher*, vol. 15, pp. 4-14.
- SHULMAN, L. S. (2004) *The Wisdom of Practice, Essays on Teaching, Learning, and Learning to Teach*. San Francisco, CA, Jossey Bass.
- SIKES, P. & POTTS, A. (2008) *Researching Education From the Inside: Investigations From Within*. Abington, Routledge.
- SIMON, M. A. & BLUME, G. W. (1994) Mathematical Modelling as a Component of Understanding Ratio-as-Measure: A Study of Prospective Elementary Teachers, *The Journal of Mathematical Behaviour*, vol. 13, pp. 183-197.
- SKEMP, R. (1976) Instrumental Understanding and Relational Understanding, *Mathematics Teaching*, vol. 77, pp. 20-26.
- SKEMP, R. R. (1989) *Mathematics in the Primary School*. Abington, Routledge.
- SMITH, J.A., FLOWERS, P., & LARKIN, M. (2009). *Interpretative phenomenological analysis: Theory, method, research*. London: Sage.
- SMITH, J. P., DISESSA, A. A., & ROSCHELLE, J. (1993). Misconceptions reconceived: A Constructivist analysis of knowledge in transition, *Journal of the Learning Sciences*, Vol 3(2), pp 115-163.
- STEINBERG, R. M., EMPSON, S. B. & CARPENTER, T. P. (2004) Inquiry into Children's Mathematical Thinking as a Means to Teacher Change, *Journal of Mathematics Teacher Education*, vol. 7, pp. 237-267.
- STRAUSS, A. L. & CORBIN, J. M. (1997) *Grounded Theory in Practice*, Thousand Oaks, CA, Sage Publications, Inc.

- SUN, X. (2011) Variation Problems and their Roles in the Topic of Fraction Division in Chinese Mathematics Textbook Examples, *Educational Studies in Mathematics*, vol. 76, pp. 65-85.
- SWAN, M. & SWAIN, J. (2010) The Impact of a Professional Development Programme on the Practices and Beliefs of Numeracy Teachers, *Journal of Further & Higher Education*, vol. 34, pp. 165-177.
- TIGHT, M. (2015). Phenomenography: the development and application of an innovative research design in higher education research. *International Journal of Social Research Methodology*, pp. 1-20.
- TUNKS, J. & WELLER, K. (2009) Changing Practice, Changing Minds, from Arithmetical to Algebraic Thinking: an Application of the Concerns-based Adoption Model (CBAM), *Educational Studies in Mathematics*, vol. 72, pp. 161-183.
- TURNER, F. (2008) Beginning Elementary Teachers Use of Representations in Mathematics Teaching, *Research in Mathematics Education*, vol. 10, pp. 209-210.
- VAN MANEN, M. (1982) Phenomenological Pedagogy, *Curriculum Inquiry*, vol. 12, pp. 283-299.
- VOUTSINA, C. (2012) Procedural and Conceptual Changes in Young Childrens' Problem Solving, *Educational Studies in Mathematics*, vol. 79, pp. 193-214.
- VYGOTSKY, L. S. (1978). *Mind in society: The development of higher psychological processes* Translated by A. R. Luria, M. Lopez-Morillas & M. Cole (with J. V. Wertsch) Cambridge, Mass.: Harvard University Press. (Original work [ca. 1930-1934])
- WARREN, E. (2009) Early Childhood Teachers' Professional Learning in Early Algebraic Thinking: A Model That Supports New Knowledge and Pedagogy, *Mathematics Teacher Education and Development*, vol. 10, pp. 30-45.
- WENGER, E. (1999) *Communities of Practice: Learning, Meaning, and Identity*. Cambridge, Cambridge University Press.
- WILLIAMS, P. (2008) *Independent Review of Mathematics in Early Years Settings and Primary Schools*. London, Department for Children, Schools and Families (DCSF).

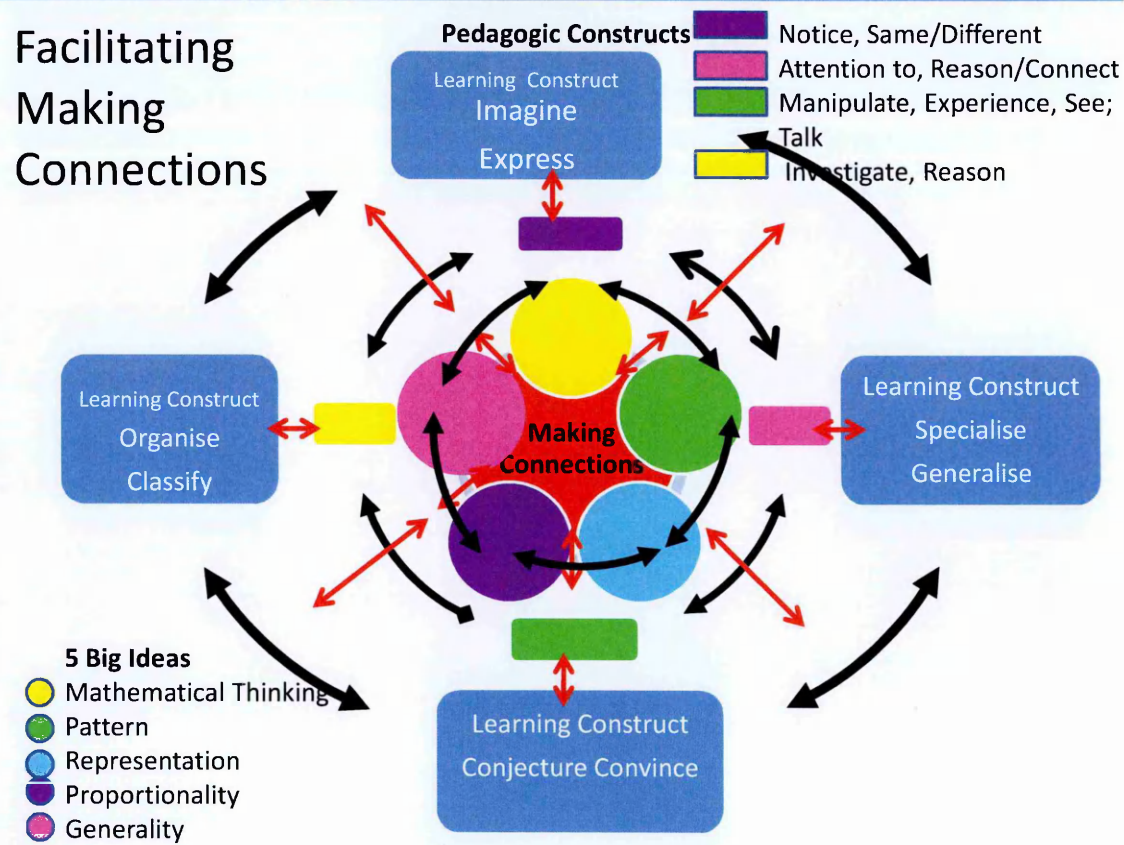
WITTGENSTEIN, L. (First published 1958) translated by ANSCOMBE G. E. M. (2010). *Philosophical investigations*. Oxford: Blackwell Ltd

ZHOU, Z., PEVERLY, S. T. & XIN, T. (2006) Knowing and Teaching Fractions: A Cross-Cultural Study of American and Chinese Mathematics Teachers, *Contemporary Educational Psychology*, vol. 31, pp. 438-457.

ZWART, R. C., WUBBELS, T., BERGEN, T. C. M. & BOLHUIS, S. (2007) Experienced Teacher Learning Within the Context of Reciprocal Peer Coaching, *Teachers & Teaching*, vol. 13, pp. 165-187.

49 982

Appendix 1a The Northampton MaST Programme Connective Model



Appendix 1b The constructs used in the Northampton MaST Programme to transfer the five big ideas into professional practice

Pedagogic Constructs	Linkages between pedagogic constructs and the five big ideas intended to support the development of teacher subject knowledge	Learning Constructs <i>Mathematical Powers</i>
Prompting thinking through specific questions: What do you notice? What is the same and what's different?	These questions promote development and application of mathematical thinking , looking for pattern , generalisation , potentially recognising proportional relationships and making mathematical connections	Engaging learners in the four pairs of learning constructs outlined below utilises and develops processes which are valuable tools to develop deep and sustainable

<p>Enabling learning through: Scaffolding</p> <p>Drawing attention to....</p> <p>Developing reasoning and making connections</p>	<p>A focus on these within teaching promote the development and application of mathematical thinking, looking for pattern, drawing attention to generality, reasoning about proportional relationships and making mathematical connections</p>	<p>learning. All have the potential to be used in conjunction with the pedagogic constructs in column 1 and provide vehicles for the application and development of the five big ideas which form the structure of the programme.</p> <p>Imagining and Expressing;</p> <p>Specialising and Generalising;</p> <p>Conjecturing and Convincing;</p> <p>Organising and Classifying</p>
<p>Stimulating learning through: Manipulate, Experience, See ; Engagement in talk (listen, analyse and discuss)</p>	<p>Involved in these constructs is the use of mathematical representations to provide access to mathematical concepts and engage in talk which develops mathematical thinking and makes mathematical connections, including those involving proportional relationships.</p>	
<p>Developing thinking through: opportunities to investigate, reason and make connections</p>	<p>These engage learners in mathematical thinking, looking for pattern, generalising and making mathematical connections.</p>	

Appendix 2 The Questionnaire

The following table outlines the questions that generated the questionnaire data for the study for each of the 3 cohorts

Question Reference	Cohort 1	Cohort 2	Cohort 3
Subject Knowledge Quantitative data (Question 1)	Has the programme met your expectations in the development of Subject Knowledge Not Met, Met, Exceeded	Has the programme met your expectations in the development of Subject Knowledge Not Met, Met, Exceeded	Has the programme met your expectations in the development of Subject Knowledge Not Met, Met, Exceeded
Personal Impact on thinking about mathematics (Question 4)	What elements of the programme, if any have had a significant impact on your thinking about mathematics? Please indicate how your thinking has changed or developed. Please name up to three aspects. If making reference to course ideas, please be specific	What elements of the programme, if any have had a significant impact on your OWN thinking and mathematical development? Please indicate how your thinking has changed or developed. Please name up to three aspects. If making reference to course ideas, please be specific.	What elements of the programme, if any have had a significant impact on your OWN thinking and mathematical development? Please indicate how your thinking has changed or developed. Please name up to three aspects. If making reference to course ideas, please be specific.
Example of response	The mathematical powers - this has made me think more about which of these I am applying when I solve mathematical problems. Skemp - relational and instrumental. This made me reflect on the way that I was taught mathematics and the way that I was teaching it.	Aspect 1: Representation. Being more aware that maths is so abstract and that there are many different representations that can help others to learn. Some are not as good as others and it's important to be aware of their strengths and weakness Aspect 2: "Linking ideas- linking ideas helps understanding rather than my previous belief that it would muddy the waters" Aspect 3:	Aspect 1: Move from instrumental teacher and thinker to relational one. Mathematical thinking is now at the very core of my teaching and thinking Aspect 2: That how concepts are connected is so important. Making many connections strengthens understanding. Aspect 3: Mathematical powers - particularly importance of conjecturing convincing. Leading into

		Before the course I never considered the difference between relational and instrumental understanding and the impact on children learning. I now feel it is vitally important for children to have a relational understanding and really understand the 'how' and 'why'.	importance of listening to children's talk and allowing them to reveal misconceptions and explore comfortably. Didn't have strong opinions before.
Impact on practice (Question 5)	What elements of the programme, if any have had significant impact on your practice? Please indicate how they have impacted on your practice and consider how your practice has changed. Please name up to three aspects. If making reference to course structure, please be specific.	What elements of the programme, if any have had significant impact on your practice? Please indicate how they have impacted on your practice and consider how your practice has changed. Please name up to three aspects. If making reference to course structure, please be specific.	What elements of the programme, if any have had significant impact on your practice? Please indicate how they have impacted on your practice and consider how your practice has changed. Please name up to three aspects. If making reference to course structure, please be specific.
Example of responses	Conjecturing and convincing - I now give children many more opportunities to discuss their ideas with me and with their peers and to convince each other where there are differences of opinion. Deeper questioning - drawing out from the children their mathematical thinking and reasoning, rather than assuming that I know what they mean! Problem solving - I provide far more opportunities for children to use and apply skills than I did	Aspect 1 Children working in pairs, small groups more than on their own Aspect 2 Generalising - children at all ages do this and as teachers we need to encourage this type of thinking. Aspect 3 Nurturing a culture of openness within the classroom whereby the children feel able to express opinions and explain without fear of being wrong: knowing that we learn from our mistakes and that a row of ticks is not necessarily all that	Aspect 1 Encouraging an understanding of mathematical concepts rather than a set of rules Aspect 2 Structural thinking - the parts relating to algebra in particular. I now spend more time considering the underpinning structures, using number properties to get children to generalise. Children generally operate at a higher level of thinking. Aspect 3 Give pupils more time and chance to express

	before.	helpful! This has grown from my own development as a mathematician and my confidence in my own mathematical understanding.	themselves, investigate, model to each other and convince others. Using the question what's same and what's different has been a successful way to give the ownership of learning to the children.
Development of Subject Knowledge (Question 6a, 6b)		Has your subject knowledge of mathematics developed? Yes No Please name up to 3 key concepts or areas of mathematics where you feel your understanding has improved.	Has your subject knowledge of mathematics developed? Yes No Please name up to 3 key concepts or areas of mathematics where you feel your understanding has improved.
Example of response		Pattern in number Ratio and Proportion Algebra	Fractions Generalisation Representation
Factors that influenced professional development (Question 7)	What key factors in terms of structure, the content or the organisation of the programme have influenced the impact on your thinking and practice?		

Appendix 3 The Interview Questions

The nature and format of the interviews was semi- structured. A common core set of questions was used for each of the group interviews and then supplementary questions were asked, as and when required.

Question 1	Have you changed? What is the nature of that change? What factors influenced the change? Was there a particular turning point, or did the change result from an accumulation of factors? Has your practice changed as much as you would have liked it to, or have inhibiting factors got in the way?
Question 2	The concept of generality, is this something you had thought of before the course, what impact has it had on your teaching? (Follow up with supplementary questions regarding the other big ideas)
Question 3	What in your opinion is depth of subject knowledge, how has your subject knowledge developed?
Question 4	What in your opinion is depth of pedagogic knowledge, how has your pedagogic knowledge developed?

Appendix 4 Example of Coding – Theme Representation

The information below represents sub coding of the original coded category of representation from across all sources. The codes that emerged are listed in the table below and the data from one category, that of “multiple”, is listed below as an example of the type of data that the study drew from. It was normal practice to record data in more than one category where it was relevant.

Name	Description	Number of Sources	Number of References
Multiple	Where there is reference to the value or use of multiple/range or variety of representations	9	40
Concrete/Manipulable	where there is reference to the use of concrete/practical representations	8	47
Visual/ Pictures	where there is reference to visual representations	7	25
Drawings	where there is reference to representing mathematics through drawing	3	6
Connections	where there is reference to representations in conjunction with making connections in mathematics	1	3
Diagrammatic/Structured	where there is reference to structured or diagrammatic representations of mathematics	4	13
Children’s own representations	where there is reference to children representing mathematics in their own way	3	17
Improved Teaching	where there is reference to use of mathematical representations and improvements to teaching	2	4
Developing Learning	Where there is specific reference to progress in learning	7	34
Teacher Subject Knowledge	where teachers make reference to how representations have improved their own subject knowledge.	6	12
Older Children	where teachers make reference to representations improving their own subject knowledge	4	12
Representing Thinking	where reference is made to mathematical representations assisting representation of or development of mathematical thinking	3	14
Right Choice	where reference is made to selecting the most appropriate representation for the mathematical concept	6	9

The section below represents the data gathered across all sources for the category of “multiple” representations” listed in the table above.

Name: Representation: Multiple Representations

<Internals\Cohort 1 Questionnaire Q4

Reference 1

Representation-studying the diversity of ways in which problems and workings can be represented and now I focus on expanding the way material is presented to the children and the encouraging the children to represent problems in

different ways for example using drawing.(representation) Identifying and describing patterns and then predicting future patterns , trying to record the pattern mathematically.

Reference 2

Representation has had a significant impact on my thinking, encouraging children to create their own representations has given me a greater insight into their understanding. Also by using multiple representations when teaching has given the children more depth of understanding and is encouraging them to make greater links between areas of mathematics.

Reference 3Representations: the need for a range of representations, and also the importance of making links between them

Reference 4

Varying the ways in which maths is represented and allowing the children to make their own representations.

Reference 5

Representation as a big idea, I have been far more aware of using different images and representations to help my pupils grasp and understand concepts and ideas

Reference 6

The course has also helped to broaden the strategies and pedagogies I use in my maths teaching. In particular it has reinforced the value of multiple representations in helping children to learn/grasp concepts and skills.

Reference 7

The big idea of representation has made me think more about how I teach new ways of working something out. Starting with the children giving it ago on their own and then discussing their methods, along with the ones set out by school has made them aware of the different ways that we can show something

Reference 8

Use of a variety of models and images to help children represent and understand

Reference 9

Equally I now provide far more opportunities for children to manipulate and experience mathematics, as I feel I can now justify (to myself and others) the value of these experiences (both from course literature, and my own observation of the impact on children's learning.)

Reference 10

Giving more consideration to how maths ideas can be represented and using different models to help children learn.

<Internals\\Cohort 1 Questionnaire Q5

Reference 1

My reception children are fascinated by maths, they are particularly fascinated by making different arrangements of amounts and looking at different representations of the same amount

Reference 2

As a teacher I take more time and care over the representations I use to demonstrate, I encourage children to use a variety of representations. As subject leader I am promoting more effective use of representations throughout the school

Reference 3

By providing different representations of fractions I have found that the children are able to apply their knowledge of fractions to many different situations.

Reference 4

Increasing the range of resources and how they are used has also increased the learning of mathematics, and the higher attainers are more willing to use resources that they may have seen as 'younger' than them before.

Reference 5

Practical resources this has allowed children to see, touch, feel, manipulate to give visual images for them to draw upon for future reference in other concepts. Offering ranges of resources and allowing children to freely chose to help explain their findings.

<Internals\\Cohort 2 Questionnaire Q4

Reference 1

Representation One of the key things I have taken away from the course is to ensure that I use a range if representations rather than my own preferred way! It is something I consider when planning and teaching. It is important for t

Reference 2

"Representation- have a fuller understanding of the benefits of representing a concept in more than one way. Before, I believed in sticking with one method so as not to cnfuse the child."

Reference 3

Representation- I am now more able to show concepts in a different way to the children. Children grasp concepts quicker and remember representations for longer.

Reference 4

activities and using a variety resources.

Reference 5

Multi representation of aspects of maths

Reference 6

The importance of a range of representations to develop conceptual understanding has played a significant part in my approach to planning and teaching Maths.

Reference 7

To use a large range of different resources and images. Pupils are able to solve tasks using a wide range of different representations and I do not model so pupils are able to express their ideas.

Reference 8

Representation - using a variety of representations to model/show mathematical concepts. Letting children represent in a way that they understand.

Reference 9

I use of a range of resources

<Internals\\Cohort 2 Questionnaire Q5

Reference 1

Representation - a range needs to be used

Reference 2

Representation - use of multiple representations

<Internals\\Cohort 3 Questionnaire Q4>

Reference 1

The representation of maths in a variety of ways including the use of resources. I used to focus on writing a lot and am now much more varied, including using resources and drawings.

Reference 2

I now consider maths to be represented in 4 main elements- words, symbols, images and equipment. I try to use a range of representations to help the children and encourage them to use a range of equipment if they are struggling with a problem.

Reference 3

Importance of multiple representations.

<Internals\\C3 Questionnaire Q5>

Reference 1

Increased use of a range of representations where possible, particularly action objects.

Reference 2

Representation - giving children a range to use to develop their mathematical thinking

Reference 3

How beneficial resources can be in representing mathematical ideas. I used to see resources as only being needed up to a certain age (KS1) and above this age, only for SEN children. I now see the importance of representing mathematical ideas in a variety of ways and encouraging the children to represent what they are thinking in their own ways. Resources are now more readily available in all classrooms within my school.

Reference 4

Providing children with the opportunities to use various representations to show their thinking

Reference 5

I consider maths as a more creative subject as you can manipulate and develop own methods to solve problems. Use of various representations has helped give children opportunity and freedom to manipulate and investigate more freely.

Reference 6

Providing ideas for using many resources has enabled our school to embrace the idea that KS2 lessons can be as resource-rich as KS1 - looking at how, for example, unifix cubes can be used to get children thinking about pattern has increased my confidence and that of others in using these resources at the upper end of the primary age range.

Reference 7

Understanding of using wider range of resources and recordings

<Internals\\Interview 1

Reference 1

When I made the changes...making sure there were far more resources and a variety of resources for the children to use all of the time, so it gave them a bit of ownership in choosing what was best for them, rather than saying we are going to use this today or we're going to use that, actually you use what you're happy with and them being able to investigate with that. I think the level of engagement that picked up from that was far greater and then you could see greater engagement.

<Internals\\Interview 3

Reference 1

Representation's been a really big thing....In our school I think teachers have realised the importance of modeling things far more with the children, giving them lots of opportunities..to see things in different ways to help them understand.

Reference 2

Representation...perhaps know a range of them as well, for one representation might not work for all, you've got to have others to fall back on as well

<Internals\\Interview 4

Reference 1

I've taught much older children where it was very kind of book orientated and I think that's (representation) made a big impact on me and looking for ways to help the children understand what the maths is and representing things in different ways, giving them choices...I think that's probably had more of an impact on me than I thought it would.

The Mathematics Specialist Teacher Programme: An Analysis of its Impact

The Research Data

The data gathered will be used both internally to improve the programme, externally to evidence impact to the Department for Education and will also form part of the Doctorate in Education which is being undertaken by the programme director Debbie Morgan, Senior Lecturer at The University of Northampton.

Ethical code

This Code is informed by the principles established in the Revised Ethical Guidelines for Educational Research (2004) issued by the British Educational Research Association (BERA)* The researchers** recognise the rights of all professional colleagues, teachers/headteachers/local authority consultants/university tutors who participate in the research to have their confidentiality protected at all times.

Voluntary informed consent will be sought before any interviews are conducted or surveys completed, with any respondent as part of the research process. In the case of school student this consent will be sought through schools and obtained in writing before any direct contact is made with the student. Parents and carers have the right to refuse participation and will not be pressured or coerced into taking part in the research. **Participants in the research have a right to withdraw from the process at any time and will be informed of this right.**

The researchers will work in accordance with Articles 3 and 12 of the United Nations Convention on the Rights of the Child and will ensure that the best interest of children is served at all times. Children will be facilitated to give informed consent and this will be in addition to the consent given by parents or carers.

The researchers are under an obligation to describe accurately, truthfully and fairly any information obtained during the course of the research. There is an obligation to incorporate accurately data collected during the course of this research into the text of any report or other publication related to the research, and to ensure that individual opinions and perceptions are not misrepresented.

The researchers will protect the sources of information gathered from interviews, surveys and other data collection methods. All data collected as part of a survey or interview will be stored securely, following the university data protection policy. All electronic data will come into one university inbox and individuals will be made anonymous before data is passed to researchers. Handwritten data will be entered anonymously onto an electronic database and names will be removed from the

original source and replaced with a code by a University of Northampton researcher or administrator. Original names and codes will be stored separately. Data collected as part of the research process will be securely maintained and will be accessible only to the researchers or administrators engaged in this project.

The researchers will communicate to external bodies the extent to which their data collection and analysis techniques and the inferences drawn from these are reliable, valid and generalisable. The researchers will report the procedures, results and analysis of the research accurately, and in sufficient detail to allow all interested parties to understand and interpret them.

The researchers have an obligation to report truthfully the findings of the research in any written or verbal report and maintain anonymity of individuals involved.

The researchers will make themselves available to discuss the procedures, conduct, or findings of the research with any party involved in the research process.

Interim reports and a research thesis will be produced and submitted for the award of Doctorate in Education to The Open University by Debbie Morgan, a Senior Lecturer at The University of Northampton and will be made available in both paper and electronic format to all participants in the research.

The researchers are obliged to communicate the findings of their research to other members of the educational research community through research seminars, conference presentation and proceedings and publication taking account of all issues of confidentiality and protection of research participants.

The researchers assert their right to participate in any publication of the research findings in academic journals or other media, which may ensue from the research.

Audio recording of interviews

Written permission will be obtained to electronically record interviews in audio format.

Audio recordings will be handled in the same way as written data. Recordings will be stored in MP3 format and names will be replaced with a code. The original recordings will be deleted from the audio recording device within one month of the recording. The participant will not be identified in any publication, sharing or presentation of the research.

I give permission to record the research interview and understand that anything I say will remain anonymous in any publication, sharing or presentation of the research.

Name

Signed.....

Date

*BERA (2004) *Revised Ethical Guidelines for Educational Researchers*. Southwell: British Educational Research Association

** The researchers refers to those individuals named as part of this research process and will include academic and support staff from the consortium of universities named below:

The University of Northampton (lead institution): The University of Bedfordshire: The University of Hertfordshire : Nottingham Trent University : The University of Derby : Bishop Grosseteste University College Lincoln

Appendix 6 Themes that emerged from the interview data

Code Name	Description	Number of Interviews	Number of references
Subject Knowledge	Where reference is made to the development of mathematics subject knowledge	8	55
Making Connections	Where reference is made to the positive benefit or importance of making connections in mathematics	8	31
Conceptual Understanding	Where reference is made to the development of conceptual understanding	8	32
Relational/ Instrumental Understanding	Where reference is made to the importance of relational understanding of mathematics and or insufficiencies of instrumental understanding	8	26
Deep	Where reference is made to deep knowledge	7	22
Structure of Programme	Where reference is made to the benefits of the structure of the programme	3	8
Big Ideas	Where explicit reference is made to the term <i>big ideas</i> or <i>bigger picture</i>	8	22
Representation	Where explicit reference is made to the use or impact of representations in teaching/learning mathematics	8	38
Generality	Where reference is made to the process of generalisation in mathematics	6	16
Proportionality	Where explicit reference is made to the big idea of proportionality	2	4
Mathematical Thinking	Where explicit reference is made to the process of mathematical thinking.	7	25
Pattern	Where reference is made to pattern in mathematics	5	7
Confidence		8	26
Talk	Where reference is made to the benefits or importance of talk in mathematics	8	36
Pedagogic Knowledge	Explicit reference to the development of pedagogic knowledge	4	8
Questioning	Reference to questioning as a teaching and learning tool	4	6

Same and Different	Where reference is made to the pedagogic construct of asking the question <i>what's the same and what's different?</i>	7	13
Pupil learning	Where reference is made to the impact of the programme on pupil learning	6	35
Practical	Reference to practical engagement in mathematics	3	4
Powers	Where reference is the learning construct of the application of mathematical powers as discussed in Chapter 1	6	38
Enjoyment/ Motivation	Where reference is made to enjoyment or motivation by pupils to engage in mathematics	4	15
Changes to Practice	Where reference is made to changes that teachers have made to their practice	7	56
Inhibiting factors	Where reference is made to factors that inhibited changes teachers wanted to make	2	3
Reasons for Change	Where indication is given as to factors which influenced teacher change	8	43
Sustainability	Where reference is made to sustainability of change	5	14
Working with Others	Where reference is made to working collaboratively with other teachers	5	24